Interest Rates, Money, and Banks in an Estimated Euro Area Model*

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Abstract

This paper estimates a medium scale macroeconomic model with costly banking for euro area data. In addition to data on measures of real activity and prices, we include data on bank credit, loan rates, and reserves for the estimation of the model with Bayesian techniques. We find that banking costs are significantly affected by credit supply and by holdings of reserves, implying non-neutrality of (high powered) money. Stochastic deviations to the demand for reserves are found to explain a large share of inflation and policy rate variations, and to significantly contribute to fluctuations output, in particular, in the last part of the sample including the recent financial crisis. In contrast, exogenous shifts in banking costs are negligible for fluctuations in real activity and prices, while they explain the largest share of variations in reserves.

JEL classification: C54, E52, E32

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1 Introduction

In this paper, we examine the informational content of banking activities and reserves for short-run macroeconomic dynamics in the Euro area. The main purpose of the analysis is to identify if and how banks' demand for high powered money matters for real activity and inflation. Macroeconomic studies on monetary policy has typically supported the view that monetary aggregates are largely irrelevant for output and inflation and can be neglected for the conduct of monetary policy (see e.g. Ireland (2004), or Woodford (2008)). While the majority of these studies have focussed on the broader monetary aggregates (such as M1 or M2), we are particularly interested in the role of central bank money. For this, we construct a medium scale macroeconomic model with costly banking, and estimate it using data on bank credit, lending rates, and reserves in addition to macroeconomic time series typically employed for estimation purposes.[2] We find that banks' holdings of reserves are non-neutral as they tend to reduce costs of loan creation. The variance of reserve holdings can, to the largest part, be explained by productivity shocks and shocks to banking costs, while the latter are irrelevant for the volatility of other macroeconomic variables. A counterfactual analysis, for which reserves are assumed not to respond to banking costs shocks, shows that real activity and inflation would have been affected by banking costs shocks, if the central bank did not fully accommodate banks' reserve demand. This effect is particularly pronounced in the last part of our sample, which covers post crisis data until 2011Q4.

The model mainly differs from standard medium scale macroeconomic models (like Smets and Wouters (2003) and (2007)) by accounting for banks intermediating funds between households and firms as well as for banks' holdings of reserves. Banks demand reserves to satisfy a minimum reserve requirement and can further ease costs of loan creation. Banks further hold government bonds, which serve as eligible assets in open market operations. The central bank controls the price of money in open market operations, which accords to the main refinancing rate of the European Central Bank (ECB). Changes in the policy rate might be incompletely passed through by banks to interest rates that are relevant for private agents saving and borrowing decisions. Given that the focus of the paper is a quantitative analysis, we apply a stylized specification of banking costs (see Curdia and Woodford (2011)), which can in principle represent different types of frictions, e.g. maturity mismatch, limited enforcement, or monitoring costs. Hence, we refrain from providing explicit microfoundations for a particular type of imperfection and take an agnostic view by considering a banking cost function with sufficient degrees of freedom for an estimation based on macroeconomic aggregates. By estimating the parameters of the banking cost function,

[2] Macroeconomic models developed for estimation purposes either neglect monetary aggregates at all, like Smets and Wouters (2007) and Christiano, Motto, and Rostagno (2014), or consider broader monetary aggregates (that corresponds to M1 or M2), like Christiano, Eichenbaum, and Evans (2005) and Aruoba and Schorfheide (2011). Curdia and Woodford (2011) build a framework with costly banking and central bank money for the analysis of unconventional monetary policy, but do not estimate the model.

the effects and the size of these costs are identified by fitting the model to data on interest rates, loans, reserves, and other macroeconomic aggregates.

We estimate the model applying Bayesian estimation techniques and Euro area data from 1981Q1 to 2011Q4, thus excluding the latest time period where reserves has been developed in a very extreme way.[3] Given that pre-crisis data suggest that banks mainly hold reserves to satisfy a minimum reserve requirement, we compare two versions of the model, where either the elasticities of banking costs with regard to reserves (and loans) are unrestricted or where we impose that reserves are irrelevant for banking costs. The latter version implies a de facto separability of (central bank) money,[4] which corresponds to the widespread view on the irrelevance of money. For the unrestricted version, we find that loans and reserves significantly affect banking costs, such that reserves are actually non-separable. We further estimate the unrestricted model for a subsample excluding the crisis period (1981Q1 to 2007Q4), to unveil if the relevance of reserves is mainly induced by the post-crisis sample. The subsample estimate also finds a significant impact of reserves on banking costs, confirming the view that non-separability of central bank money has been a structural feature of the European banking system.

Overall, the performances of both versions of the model, i.e. the unrestricted model and the restricted model, are very similar and are comparable to the results in Smets and Wouters (2003) with regard to the non-financial variables and shocks. In terms of standard deviations and output correlations both model versions are well in line with the data, except for the observed positive correlation between reserves and real gdp, which the version with the restricted banking cost parameter greatly fails to reproduce. The decomposition of individual time series further shows that in both versions productivity shocks contribute to a larger extent to the variation of reserves, loans, and the lending rate than monetary policy shocks, i.e. shocks to a feedback rule for the policy rate. Banking costs are negligible for macroeconomic fluctuations, except for the volatility of reserves for which they are relevant in the unrestricted version. For the restricted version, we find that stochastic deviations from the minimum reserve requirement, i.e. money demand shocks, are irrelevant for dynamics of macroeconomic aggregates except for reserves, revealing that money is de facto separable in the version (S). For the unrestricted version, we find that stochastic deviations from banks' money demand contribute more to fluctuations in most macroeconomic series than interest rate shocks, and that they have particularly been relevant for changes in output in the last part of the sample including the recent financial crisis. We view this result, which is consistent with the view of that central bank money is non-separable (NS), as suggestive for substantial

[3] In 2012, the ECB introduced some extraordinary monetary operations, which have lead to an extreme upward shift in total reserves. These policy measures are not taken into account in the model and are beyond the scope of this paper.

[4] Due to the property that reserves affect the balances sheets of banks and the central bank, reserves are structually not separable from the real allocation and prices. However, our estimations show that are actually no measurable effects of changes in reserves, such that reserves are de facto separable.

information on monetary policy contained in reserve data, which cannot solely be captured by a rule for the monetary policy rate.

The remainder is organized as follows. Section 2 presents the model. Section 3 discusses some equilibrium properties. In Section 4 we describe the calibration and estimation of the model. In Section 5 we present the quantitative results. Section 6 concludes.

2 The model

Following Smets and Wouters (2003), we model the Euro Area as a closed economy. The economy consists of five distinct sectors: The household sector, the production sector and fiscal policy are close to standard specifications, while the financial intermediation as well as the central bank activities are augmented and modified to allow for a meaningful interaction between banks and the central bank. The central bank sets the price of money in open market operations. Banks receive deposits from households and supply loans to firms, while operating under a balance sheet constraint and costs associated with bank lending. They further hold reserves and bonds issued by the government, which serve as collateral for reserves in open market operations. Firms rely on external funds for working capital, as in Christiano, Eichenbaum, and Evans (2005), while we assume that households cannot directly lend to firms. Households hold deposits, which provide transaction services, and are assumed to have access to a full set of nominally state contingent claims.[5] The government issues state-contingent multiperiod bonds, purchases goods, and raises lump-sum taxes.

2.1 Households

There is a continuum of infinitely lived households indexed with $i \in [0,1]$. Households have identical preferences and potentially different asset endowments. Household utility increases with consumption and decreases with working time. We further assume that beginning-of-period holdings of deposits $D_{i,t-1}$ at commercial banks provide utility, which serves as a short-cut for modelling transaction services of deposits and thus for considering deposits as a component of broader monetary aggregates. Household i maximizes the expected sum of a discounted stream of instantaneous utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t u(c_{i,t}, c_{t-1}, n_{i,t}, D_{i,t-1}/P_t), \tag{1}$$

where E_0 is the expectations operator, $\beta \in (0,1)$ a discount factor, and ξ_t a time preference shock. Instantaneous utility depends on individual consumption $c_{i,t}$, working time $n_{i,t}$, the real value of bank deposits $d_{i,t} = D_{i,t}/P_t$, where P_t denotes the price of the wholesale good, and c_t aggregate consumption; the latter affecting individual utility via external habits. We apply the following instantaneous utility function: $u_{i,t} = \frac{1}{1-\sigma}\left(c_{i,t} - hc_{t-1}\right)^{1-\sigma} + \varrho \frac{1}{1-\varphi}\left(d_{i,t-1}\pi_t^{-1}\right)^{1-\varphi} - \nu_t \frac{1}{1+\upsilon} n_{i,t}^{1+\upsilon}$,

[5] Market completeness is assumed to facilitate aggregation and comparisons to related studies.

such that $u_{i,ct} = (c_{i,t} - hc_{t-1})^{-\sigma}$, $u_{i,dt} = \varrho\pi_t^{-1}(d_{i,t-1}/\pi_t)^{-\varphi}$ and $u_{i,nt} = -\nu_t n_{i,t}^{\upsilon}$, where $\sigma > 0$, $\varphi > 0$, $\upsilon \geq 0$, and $\varrho \geq 0$, $\pi_t = P_t/P_{t-1}$ denotes the inflation rate, and ν_t a labor supply shock. Household i supplies differentiated labor services at the nominal wage rate $W_{i,t}$, invests in deposits, and trades state contingent claims $S_{i,t}$:

$$(D_{i,t}/R_t^d) - D_{i,t-1} + E_t[\varphi_{t,t+1}S_{i,t+1}] - S_{i,t} + P_t c_{i,t} \leq W_{i,t}n_{i,t} + P_t pr_{i,t} + P_t\tau_{i,t} + P_t\tau_{i,t}^m, \tag{2}$$

where R_t^d denotes the rate of return on deposits, $\varphi_{t,t+1}$ a stochastic discount factor, $\tau_{i,t}$ a lump-sum tax, and $pr_{i,t}$ collects profits from firms, retailers, and banks. Household $i's$ borrowing is restricted by $D_{i,t} \geq 0$ and $\lim_{s\to\infty} E_t\varphi_{t,t+s}S_{i,t+s+1} \geq 0$. Maximizing the objective (1) subject to (2) and the borrowing constraints, for given initial values $D_{i,-1} > 0$, $S_{i,0}$, $c_{-1} > 0$ leads to first order conditions for consumption, deposits, and contingent claims, which can be summarized as $\xi_t u_{c,i,t} = \lambda_{i,t}$,

$$1/R_t^d = \beta E_t\left[\frac{1}{\pi_{t+1}}\frac{\xi_{t+1}u_{c,i,t+1}}{\xi_t u_{c,i,t}}\left(1 + \frac{u_{d,i,t+1}}{u_{c,i,t+1}}\right)\right], \tag{3}$$

$$\varphi_{t,t+1} = \beta\frac{1}{\pi_{t+1}}\frac{\xi_{t+1}u_{c,i,t+1}}{\xi_t u_{c,i,t}}, \text{ where } R_t = 1/E_t\varphi_{t,t+1}, \tag{4}$$

and (2) holding with equality as well as the transversality conditions. A comparison of (3) and (4) shows that the deposit rate R_t^d tends to be smaller than the risk-free rate R_t, as deposits increase utility. Combining (3) and (4) leads to a version of deposit demand, $1 = E_t[R_t^d\varphi_{t,t+1}(1 + u_{d,i,t+1}/u_{c,i,t+1})]$, which accords to a conventional demand condition for an assets that provide transaction services (except for the deposit rate R_t^d). It implies that the demand for real deposits tends to decrease with the spread between the risk-free rate and the deposit rate. In contrast to the common approach of specifying monetary policy in macroeconomic models (see e.g. Smets and Wouters (2007)), we do not assume that the central bank is able to control the risk-free rate directly. Instead, the central bank sets the price of money in open market operations (which accords to the ECB's main refinancing rate), while other interest rates (including R_t^d and R_t) are endogenously determined. Further note that time preference shocks ξ_t, which differ from ad-hoc risk premium shocks that are introduced by Smets and Wouters (2007) to account for differences between the marginal rate of intertemporal substitution and the monetary policy rate, apply to all intertemporal decisions and prices (see also Sections 2.2 and 2.3).

We assume that households monopolistically supply differentiated labor services $n_{i,t}$, which are transformed into aggregate working time n_t as $n_t^{1-1/\varepsilon_n} = \int_0^1 n_{i,t}^{1-1/\varepsilon_n}di$, where $\varepsilon_n > 1$ is the elasticity of substitution between differentiated labor services. Cost minimization then leads to the following labor demand

$$n_{i,t} = (W_{i,t}/W_t)^{-\varepsilon_n}n_t, \tag{5}$$

where $W_t^{1-\varepsilon_n} = \int_0^1 W_{i,t}^{1-\varepsilon_n}di$ and W_t denotes the aggregate wage rate. We assume that nominal wages $W_{i,t}$ are set in staggered way, as in Erceg et al. (2000). In any period, only a constant fraction $1-\varsigma$ (where $\varsigma \in (0,1)$) of households receives a random signal allowing household i to re-optimize its nominal wage. The remaining fraction adjusts the nominal wage rate mechanically with the past inflation rate π_{t-1} , such that $W_{i,t} = \pi_{t-1}W_{i,t-1}$ in this case. If household i is allowed to change its wage rate in period t, it maximizes (1) subject to labor demand (5), leading to the following first order condition for the wage rate $\widetilde{W}_t$

$$E_t \sum_{s=0}^{\infty} \beta^s \varsigma^s \left[\frac{\xi_{t+s} u_{c,i,t+s}}{\xi_t u_{c,i,t}} n_{i,t+s} \left(\frac{(\Pi_{k=1}^s \pi_{t+k-1}) \widetilde{W}_t}{P_{t+s}} - \frac{\varepsilon_n}{\varepsilon_n - 1} mrs_{i,t+s} \right) \right] = 0, \tag{6}$$

where $mrs_{i,t}$ denotes the household i's marginal rate of substitution between consumption and leisure, $mrs_{i,t} = -u_{i,nt}/u_{c,i,t}$. Using $(\Pi_{k=1}^s \pi_{t+k-1}) \widetilde{W}_t / P_{t+s} = (\pi_t/\pi_{t+s})\widetilde{w}_t$ where $\widetilde{w}_t = \widetilde{W}_t/P_t$, (6) can be written as $f_t^1 = f_t^2$, where $f_t^1 = \widetilde{w}_t \xi_t u_{c,i,t} \left(w_t/\widetilde{w}_t\right)^{\varepsilon_n} n_{i,t} + E_t \beta\varsigma[(\pi_{t+1}/\pi_t)\left(\widetilde{w}_{t+1}/\widetilde{w}_t\right)]^{\varepsilon_n - 1} f_{t+1}^1$ and $f_t^2 = \epsilon_{w,t}\nu\mu_w\xi_t \left(w_t/\widetilde{w}_t\right)^{(1+\upsilon)\varepsilon_n} n_{i,t}^{(1+\upsilon)} + \beta\varsigma[(\pi_{t+1}/\pi_t)\left(\widetilde{w}_{t+1}/\widetilde{w}_t\right)]^{(1+\upsilon)\varepsilon_n} f_{t+1}^2$, where $\mu_w = \varepsilon_n/(\varepsilon_n - 1)$ and we defined $\epsilon_{w,t} = v_t/v$ with v as the mean of v_t and $\epsilon_{w,t}$ a shock that is equivalent to a wage-markup shock (see Chari, Kehoe, and McGrattan (2009) for a critical discussion on this issue). As trade in contingent assets implies that (the growth rate of) the marginal utility of consumption is the same across households (see 4), any household who is permitted to optimize chooses the same nominal wage rate $\widetilde{W}_t$. The aggregate real wage rate $w_t = W_t/P_t$ then evolves according to $w_t = [\varsigma w_{t-1}^{1-\varepsilon_n} \left(\pi_t/\pi_{t-1}\right)^{\varepsilon_n - 1} + (1-\varsigma)\widetilde{w}_t^{1-\eta}]^{1/(1-\varepsilon_n)}$.

2.2 Production

The production sector consists of monopolistically competitive intermediate goods producing firms, monopolistically competitive retailers, and perfectly competitive bundlers who supply the final wholesale good.

There is a continuum of monopolistically competitive intermediate goods producing firms. Firm $j \in [0,1]$ produces intermediate goods $y_{j,t}^m$ with labor, which is hired from households, and with their own stock of capital $k_{j,t}$. Individual intermediate goods $y_{j,t}^m$ are sold at the price $Z_{j,t}$ to retailer k, which demand individual intermediate goods according to $y_{j,t}^m = \left(Z_{j,t}/Z_t\right)^{-\varepsilon_{m,t}} y_{k,t}$, where $\varepsilon_{m,t}$ denotes a random substitution elasticity that serves as a cost-push shock. The production technology is identical for all firms j and exhibits standard neoclassical properties: $y_{j,t}^m = a_t n_{j,t}^{\alpha} k_{j,t-1}^{1-\alpha}$, where $\alpha \in (0,1)$ and a_t is a random productivity level with mean one. A firm j accumulates physical capital $k_{j,t}$ by investing $x_{j,t}$ and subject to adjustment costs $\Gamma_{X,t} = \Gamma_X \left(x_{j,t}/x_{j,t-1}\right)$ associated with changes in investment

$$k_{j,t} - (1-\delta)k_{j,t-1} = \epsilon_{x,t} \left(1 - \Gamma_{X,t}\right) x_{j,t}, \tag{7}$$

where $\Gamma_{X,t} = \frac{\gamma_X}{2}(\frac{x_{j,t}}{x_{j,t-1}} - 1)^2$ with $\gamma_X > 0$ and $\delta \in (0,1)$ denotes the depreciation rate and $\epsilon_{x,t}$

an investment-specific technology shock. Firms have access external funds via one-period risk free bank loans $L_{j,t}$ at the current price $1/R_t^L$. For simplicity, we assume that firm owners receive claims v_t^f on current period profits (including repayment of previous period debt) at the beginning of each period, such that v_t^f is given by

$$P_t v_t^f = Z_{j,t} a_t n_t^{\alpha} k_{j,t-1}^{1-\alpha} - P_t w_t n_{j,t} + \left(L_{j,t}/R_t^L\right) - P_t x_{j,t} - L_{j,t-1}, \tag{8}$$

where $Z_{j,t}$ denotes the price of the intermediate good. Demand for external funds is then induced by assuming that wages have to be paid on workers' banking accounts before goods are sold. Firm $j's$ current period demand for one-period loans $L_{j,t}$ from banks thus satisfies:

$$L_{j,t}/R_t^L \geq w_t n_{j,t}. \tag{9}$$

We assume that, in equilibrium, firms fully repay one unit of currency per unit of loan in the subsequent period, such that R_t^L denotes a risk-free rate of return on loans. We assume that firm j maximizes the present value of dividends, $\max E_t \sum_{k=0}^{\infty} \phi_{t,t+k} v_{t+k}^f$, s.t. $y_{j,t}^m = \left(Z_{j,t}/Z_t\right)^{-\varepsilon_{m,t}} y_{k,t}$, (7)-(9), and a no-Ponzi game condition, where $\phi_{t,t+k} = \varphi_{t,t+1}\pi_{t+1} \cdot \varphi_{t+1,t+2}\pi_{t+2} \cdot \ldots \cdot \varphi_{t+k-1,t+k}\pi_{t+k}$ denotes the firms' stochastic discount factor (see 2), given $k_{j,-1} > 0$ and $x_{j,-1} > 0$. The first order conditions for labor and loans are

$$\left(mc_{j,t}/\mu_{p,t}\right) \alpha a_t n_{j,t}^{\alpha-1} k_{j,t-1}^{1-\alpha} = w_t R_t^L / R_t, \tag{10}$$

$$l_{j,t}/R_t^L = w_t n_{j,t}, \text{ if } R_t^L > R_t \text{ or } l_{j,t}/R_t^L \geq w_t n_{j,t}, \text{ if } R_t^L = R_t, \tag{11}$$

where we defined $mc_{j,t} = Z_{j,t}/P_t$ and $\mu_{p,t} = \frac{\varepsilon_{m,t}}{\varepsilon_{m,t}-1}$. Labor demand (10) is effectively altered by the working capital constraint (9), if the lending rate R_t^L exceeds the risk-free rate R_t, which will be the case in equilibrium (mainly) due to positive banking costs that banks pass to the lending rate. The working capital constraint (11) will thus be binding throughout the analysis. The first order conditions for investment expenditures and physical capital are further given by

$$1 = q_{j,t}\epsilon_{xt}\left(1 - \Gamma_{X,t} - \Gamma'_{X,t}\frac{x_{j,t}}{x_{j,t-1}}\right) + E_t\left[\phi_{t,t+1} q_{j,t+1}\epsilon_{x,t+1}\Gamma'_{X,t+1}\left(\frac{x_{j,t+1}}{x_{j,t}}\right)^2\right], \tag{12}$$

$$q_{j,t} = E_t[\phi_{t,t+1} q_{j,t+1}(1-\delta)] + E_t[\phi_{t,t+1}\left(mc_{j,t+1}/\mu_{p,t+1}\right)(1-\alpha) n_{j,t+1}^{\alpha} k_{j,t}^{-\alpha}], \tag{13}$$

where q_t denotes the standard Tobin's q. Given that all intermediate goods producing firms behave in an identical way, aggregate supply simply equals $y_t^m = y_{j,t}^m$.

A monopolistically competitive *retailer* $k \in [0,1]$ buys intermediate goods $y_{j,t}^m$ at the price $Z_{j,t}$, combines them to the retail good $y_{k,t}$ according to $(y_{k,t})^{\frac{\varepsilon_{m,t}-1}{\varepsilon_{m,t}}} = \int_0^1 \left(y_{j,t}^m\right)^{\frac{\varepsilon_{m,t}-1}{\varepsilon_{m,t}}} dj$, and sells it at the price $P_{k,t}$ to perfectly competitive *bundlers*. The latter bundle the goods $y_{k,t}$ to the final consumption good y_t with the technology, $y_t^{\frac{\varepsilon-1}{\varepsilon}} = \int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}} dk$, where $\varepsilon > 1$ is the elasticity of substitution and the cost minimizing demand for $y_{k,t}$ is $y_{k,t} = (P_{k,t}/P_t)^{-\varepsilon} y_t$. A fraction $1-\phi$ of the retailers set their price in an optimizing way. The remaining fraction $\phi \in (0,1)$ of retailers adjust the price according to partial indexation to the previous period inflation rate π_{t-1}, $P_{k,t} = \pi_{t-1}^{\iota} P_{k,t-1}$. The problem of a price adjusting retailer is

$$\max_{\widetilde{P}_{k,t}} E_t \sum_{s=0}^{\infty} \phi^s \beta^s \phi_{t,t+s} \left(\frac{\left(\Pi_{k=1}^{s} \pi_{t+k-1}^{\iota}\right) \widetilde{P}_{k,t}}{P_{t+s}} - mc_{t+s} \right) y_{k,t+s}, \tag{14}$$

where $mc_t = Z_t/P_t$ and $Z_t^{1-\varepsilon_{m,t}} = \int_0^1 Z_{j,t}^{1-\varepsilon_{m,t}} dj$. The first order condition (14) can equivalently be written as $\tilde{Z}_t = \frac{\varepsilon}{\varepsilon-1} Z_t^1/Z_t^2$, where $\tilde{Z}_t = \widetilde{P}_t/P_t$, $Z_t^1 = \xi_t c_t^{-\sigma} y_t mc_t + \phi\beta E_t \left(\pi_{t+1}/\pi_t^{\iota}\right)^{\varepsilon} Z_{t+1}^1$ and $Z_t^2 = \xi_t c_t^{-\sigma} y_t + \phi\beta E_t \left(\pi_{t+1}/\pi_t^{\iota}\right)^{\varepsilon-1} Z_{t+1}^2$. With perfectly competitive bundlers and the homogenous bundling technology, the price index P_t for the final consumption good satisfies $P_t^{1-\varepsilon} = \int_0^1 P_{k,t}^{1-\varepsilon} dk$. Hence, we obtain $1 = (1-\phi)\,\tilde{Z}_t^{1-\varepsilon} + \phi \left(\pi_t/\pi_{t-1}^{\iota}\right)^{\varepsilon-1}$, where $\iota \in [0,1]$ measures the degree of indexation. In a symmetric equilibrium, $y_{j,t}^m = y_{k,t}$ will hold and thus $y_t = a_t n_t^{\alpha} k_{t-1}^{1-\alpha}/s_t$, where $s_t = \int_0^1 \left(P_{k,t}/P_t\right)^{-\varepsilon} dk$ and $s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1} \left(\pi_t/\pi_{t-1}^{\iota}\right)^{\varepsilon}$ given s_{-1}.

2.3 Banks

The basic role of banks in this model is to intermediate funds between households, firms, and the public sector. There is a continuum of perfectly competitive financial intermediaries, i.e. commercial banks. We account for the fact that in each period banks have to satisfy a balance sheet constraint and a minimum reserve requirement. We further consider real resource costs stemming from the origination and the supply of loans to firms. Following Curdia and Woodford (2011), we allow for these banking costs to be increasing in the amount of loans and to be decreasing in the amount of reserves that are available for the liquidity management of credit supply. They receive deposits from household $D_t = \int D_{i,t} di$, supply loans $L_t = \int L_{j,t} dj$, and further hold reserves M_t and multiperiod government bonds B_t, which are traded at a price q_t^B in period t and deliver a payoff p_{t+1}^B in period $t+1$ (see Section 2.4). The bank balance sheet constraint, which requires that they accept deposits to the amount that equals the expected payoffs from assets (see Curdia and Woodford (2011)), thus reads:

$$D_t = M_t + E_t p_{t+1}^B B_t + L_t. \tag{15}$$

Before banks enter the asset market, they exchange eligible assets against reserves with the central bank in open market operations. Banks use government bonds as collateral to get additional reserves $I_t = M_t - M_{t-1}$ from the central bank. We assume (without modeling) that eligible assets are abundantly available by banks, i.e. that $I_t \leq B_t/R_t^m$, where $R_t^m > 1$ denotes the main refinancing rate that serves as the policy instrument. To satisfy a minimum reserve requirement, banks have to hold reserves as a minimum a fraction of their deposits:

$$M_t \geq \mu D_t. \tag{16}$$

where we allow for time-varying disturbances to minimum reserve ratio (see below). We specify costs of banking activities in a stylized way. While lacking an explicit microfoundation, we introduce a functional form of real resource costs that can be identified by estimating few parameters.[6] We assume that banks face real resource costs when they originate and fund loans to firms. Following Curdia and Woodford (2011), we assume that these costs Ξ_t are increasing in the amount of loans, $\Xi_{l,t} \geq 0$, and decreasing in the amount of reserves held by banks, $\Xi_{m,t} \leq 0$. In particular, we assume that total reserves $M_{t-1} + I_t$ net of required reserves reduce banks' costs:

$$\Xi_t = \Xi(L_t/P_t, (M_{t-1} + I_t - \mu_t D_{t-1})/P_t), \tag{17}$$

Throughout the analysis, we will apply the specific form: $\Xi_t = \zeta_t\left([L_t/P_t]/[(M_{t-1} + I_t - \mu_t D_{t-1})/P_t]^{\omega}\right)^{\eta_{rc}}$, where $\omega \geq 0$, $\eta_{rc} \geq 0$, and the stochastic term ζ_t serves as a shock to banking costs. Given that bonds are discounted at the rate R_t^m in open market operations, acquisition of reserves I_t is associated with costs $I_t\left(R_t^m - 1\right)$. Real profits of a bank v_t^I are thus given by

$$P_t v_t^I = (D_t/R_t^d) - D_{t-1} - q_t^B B_t + p_t^B B_{t-1} - (L_t/R_t^L) + L_{t-1} - M_t + M_{t-1} - I_t\left(R_t^m - 1\right) - P_t\Xi_t, \tag{18}$$

where $m_t = M_t/P_t$, $i_t = i_t/P_t$, and $l_t = L_t/P_t$, and q_t^B denotes the end-of-period price (or issuance price) of government bonds. Banks maximize the sum of discounted profits, where they take the balance sheet constraint (15) as well as the minimum reserve requirement (16) into account: $\max E_t \sum_{k=0}^{\infty} \phi_{t,t+k} v_{t+k}^I$, s.t. (15)-(18), and a no-Ponzi game condition $\lim_{s\to\infty} E_t\phi_{t,t+s}D_{i,t+s} \geq 0$ as well as $L_t \geq 0$, $B_t \geq 0$, and $M_t \geq 0$. The first order conditions with regard to deposits, bonds, loans, money holdings, and additional reserves I_t, which can be combined to

$$1/R_t^d = 1 - E_t\left(R_{t+1}^m - 1\right)\left(1 - \mu_{t+1}\right)\varphi_{t,t+1}, \tag{19}$$

$$1/E_t R_{t+1}^b = 1/R_t^d - E_t\left(R_{t+1}^m - 1\right)\mu_{t+1}\varphi_{t,t+1}, \tag{20}$$

$$1/R_t^L = 1/R_t^d - E_t\left(R_{t+1}^m - 1\right)\varphi_{t,t+1}\mu_{t+1} - \Xi_{l,t}, \tag{21}$$

$$\Xi_{m,t} = 1 - R_t^m + \eta_t, \tag{22}$$

(where $\varphi_{t,t+1} = \phi_{t,t+1}\pi_{t+1}^{-1}$) as well as (15), and the complementary slackness conditions

$$\eta_t\left(m_t - \mu_t d_{t-1}\pi_t^{-1}\right) = 0,\ \eta_t \geq 0,\ m_t - \mu_t d_{t-1}\pi_t^{-1} \geq 0, \tag{23}$$

[6] Alternative approaches to specify financial intermediation and associated imperfecions in a more rigorous way, like Gertler and Kiyotaki (2010) and (2013), are theoretically more appealing, but are less suited for the quantitative analysis of banks' reserve demand.

where η_t denotes the multiplier on the minimum reserve requirement (16) and R_t^b is defined as the one-period rate of return on state contingent government bonds, $R_t^b = p_t^B / q_{t-1}^B$. Condition (19) relates the rate of return on deposits to the expected policy rate R_t^m, taking into account the costs induced by required reserves. The return on risk-free government bonds (see 20) relates to the return on deposits and to the marginal costs of holding deposits. The return on loans additionally accounts for the marginal effects of loans on the banking costs (see 21). Finally, banks' demand for reserves satisfies (22), which relates the payoff from holding reserves (via reductions of the banking costs) to the policy rate, i.e. the costs of acquiring reserves in open market operations, and to the multiplier on the minimum reserve requirement. The constraints (15), (16), and the optimality conditions (19)-(22), describe the banks' behavior.

We will consider two scenarios that differ with regard to the elasticity ω of the banking costs with respect to reserves that exceed the minimum reserve requirement. When the elasticity equals zero, $\omega = 0 \Rightarrow \Xi_{m,t} = 0$, banks will only hold required reserves, as (22) implies $\eta_t = R_t^m - 1 > 0$ and thus money demand is given by $m_t = \mu\epsilon_{m,t} \cdot d_{t-1}\pi_t^{-1}$ (see 23). If, however, the elasticity ω is positive, banks will hold more reserves than required by (16), such that the latter is slack and money demand is in principle characterized by (see 22). In this case, we consider a disturbance term $\epsilon_{m,t}$, instead of the multiplier η_t, which corresponds to the exogenous shifts in the reserve requirement ration μ_t when $\omega = 0$ (see 16), such that money demand for $R_t^m > 1$ reads

$$m_t = \begin{cases} \left[\omega\eta_{rc}\zeta_t l_t{}^{\eta_{rc}} / \left(R_t^m - 1 - \epsilon_{m,t}\right)\right]^{1/(1+\omega\eta_{rc})} + \mu d_{t-1}\pi_t^{-1}, & \text{if } \omega > 0 \text{ (version } NS) \\ \mu\epsilon_{m,t} \cdot d_{t-1}\pi_t^{-1}, & \text{if } \omega = 0 \text{ (version } S) \end{cases} \tag{24}$$

where we used that $m_t = i_t - m_{t-1}\pi_t^{-1}$ holds. Throughout the remainder of the analysis, we will refer to shocks $\epsilon_{m,t}$ as money demand shocks, even though they might also be interpretable as monetary policy shocks. Further note that the reserve requirement ratio μ will be held constant at 2% consistent with the data.

2.4 The government

The government raises lump-sum taxes τ_t and purchases goods g_t. It further issues nominal debt as perpetuities with coupons payments that decay exponentially at the rate $\rho \in [0, 1]$, which exhibit a (real) state-contingent beginning-of-period price p_t^B. Since bonds issued in period $t - s$ are equivalent to ρ^s bonds issued in t, we assume – without loss of generality – that all long-term debt are of one type (which implies that the government redeems all old bonds in each period). The price of a perpetuity issued in period t is q_t^B, while it pays out $1 + \rho q_{t+1}^B$ units of currency in period $t + 1$, such that $p_t^B = 1 + \rho q_t^B$. Let B_t^T denote the total stock of newly issued bonds, which is either held by banks or the central bank: $B_t^T = B_t + B_t^c$. The flow budget constraint of the government can be written as

$$q_t^B B_t^T + P_t sp_t = \left(1 + \rho q_t^B\right) B_{t-1}^T, \quad \text{with } p_0^B B_{-1}^T > 0, \tag{25}$$

or in real terms $q_t^B b_t^T + sp_t = \left(1 + \rho q_t^B\right) b_{t-1}^T \pi_t^{-1}$, where $b_t^T = B_t^T / P_t$ and sp_t denotes real surpluses $sp_t = \tau_t + \tau_t^m - g_t$, given $b_{-1}^T \geq 0$, and τ_t^m denotes central bank transfers. The flow budget constraint (25) implies that the government is perfectly committed to pay the coupon ρ in all periods and states. The government raises the primary surplus with the current market value of outstanding debt. For simplicity, we define $\widetilde{\tau}_t$ as total revenues from taxation and from central bank transfers, $\widetilde{\tau}_t = \tau_t + \tau_t^m$, and assume that the government controls $\widetilde{\tau}_t$ according to the following feedback rule in terms of deviations from steady state values (which are denoted without time indices):

$$\widetilde{\tau}_t - \widetilde{\tau} = g_t - g + \rho_{\tau b} \left(p_t^B b_{t-1}^T \pi_t^{-1} - \overline{pb}^T \pi^{-1} \right) + \rho_{\tau y} \left(y_t - y \right), \tag{26}$$

where $\rho_{\tau y} \geq 0$. We assume that the government targets a long-run real value for public debt $\overline{pb}^T$ that has to equal its long-run equilibrium value $\overline{pb}^T = p^B b^T$, for which the government chooses long-run transfers τ in accordance with (25). We further restrict our attention to sufficiently large values for $\rho_{\tau b}$ to ensure that intertemporal solvency is satisfied in all states and periods. To complete the specification of fiscal policy, we assume that the sequence of government spending $\{g_t\}_{t=0}^{\infty}$ is stochastic and evolves according to $g_t = \rho_g g_{t-1} + (1 - \rho_g) g + \varepsilon_{g,t}$, where $g > 0$, $\rho_g \in (0, 1)$, and $\varepsilon_{g,t}$ is i.i.d. with mean zero. Hence, lump-sum transfers are set by the government to satisfy (26) for given expenditures and central bank transfers.

2.5 The central bank

The central bank supplies money in open market operations $M_t = \int_0^1 M_{i,t} di$, such that newly issued money satisfy $I_t = M_t - M_{t-1}$, for which the central bank receives government bonds B_t^c. Hence, in open market operations t the central bank receives $I_t R_t^m$ units of bonds for which it supplies I_t units of money, such that its budget constraint reads

$$q_t B_t^c - p_t^B B_{t-1}^c + P_t \tau_t^m = \left(M_t - M_{t-1} \right) R_t^m. \tag{27}$$

where B_t^c denotes the stock of government bonds held by the central bank. In accordance with central bank practice, the central bank transfers its interest earnings from issuing money via repos and from holding interest bearing assets: $P_t \tau_t^m = E_t p_{t+1}^B B_t^c - q_t B_t^c + \left(R_t^m - 1 \right) \left(M_t - M_{t-1} \right)$. In principle, transfers can be negative when a fall in bond prices exceeds the interest earnings from money supply.[7] Substituting out transfers in (27), central bank bond holdings evolve according to

[7] See (Hall and Reis, 2012) for a comprehensive discussion of central bank solvency.

$E_t p^B_{t+1} B^c_t - p^B_t B^c_{t-1} = M_t - M_{t-1}$, and, by assuming that initial stocks satisfy $p^B_0 B^c_{-1} = M_{-1}$,

$$E_t p^B_{t+1} B^c_t = M_t, \tag{28}$$

which corresponds to the banks' balance sheet constraint (15). For the policy rate R^m_t, which in the Euro Area accords to the main refinancing rate, we apply a conventional specification and consider a simple feedback rule, which describes how the central bank adjusts the policy rate in response to changes in its own lags, in inflation, and the output-gap as a measure for real activity:

$$R^m_t = \left(R^m_{t-1}\right)^{\rho_R} \left(R^m\right)^{1-\rho_R} \left(\pi_t/\pi\right)^{\rho_\pi(1-\rho_R)} (y_t/y)^{\rho_y(1-\rho_R)} \exp \varepsilon_{r,t}, \tag{29}$$

where $R^m > 1$, $\rho_R \geq 0$, $\rho_\pi \geq 0$, and $\rho_y \geq 0$, and the $\varepsilon'_{r,t}s$ are normally and i.i.d. with $E_{t-1}\varepsilon_{r,t} = 0$. As common in the literature, we assume that the central bank chooses the inflation target $\overline{\pi}$, which has to be equal to the long-run equilibrium inflation rate π, for which the central bank sets its instruments in a consistent way.

3 Equilibrium

In this Section, we describe some main equilibrium properties of the model. In equilibrium, all markets clear and households as well as intermediate goods producing firms behave in an identical way (see Appendix A.1 for a full set of equilibrium conditions). Throughout the analysis, we will restrict our attention to equilibria where the working capital constraint of firms (9) is binding, which requires $R^L_t > R_t$. We then consider two versions, which differ with regard to the multiplier on the minimum reserve requirement (16), which is binding ($\eta_t > 0$) in version S and slack ($\eta_t = 0$) in version NS. The definitions of rational expectations equilibria for both versions are given in Appendix A.1.

Non-separability of central bank money When the elasticity of banking costs with respect to money demand $\omega\eta_{rc}$ is zero, which we will imposed in the estimations by $\omega = 0$, banks' holdings of reserves simply satisfy $m_t = \mu\epsilon_{m,t} d_{t-1}\pi_t^{-1}$ for $R^m_t > 1$ (see 24), while they are otherwise irrelevant for the decision of banks. Hence, reserves can Separately be examined from the equilibrium real allocation and the associated price system, such that we refer to this as the S version of the model. If, however, the elasticity ω is strictly positive, reserves are Not Separable from the equilibrium real allocation and the associated price system, why we then refer to the NS version. Actually, when we estimate the unrestricted version of the model, we find that the elasticity $\omega\eta_{rc}$ is strictly positive confirming that reserves are indeed non-separable. The (non-)separability of reserves in the long-run equilibrium is analyzed in Appendix A.2, where we further show that changes in the the minimum reserve requirement are not neutral in both versions.

Monetary transmission In models with frictionless financial markets, the central bank is typically assumed to be able to control the risk-free nominal interest rate R_t, which – in real terms – governs the marginal rate of intertemporal substitution, $\beta E_t[\xi_{t+1}u_{c,t+1}/(\xi_t u_{c,t})]$. In this model, the central bank is – in accordance with the ECB practice – assumed to control the price of money in open market operations, which – via profit maximizing behavior of competitive banks – affects the interest rates on deposits, loans, and government bonds. The pass-through of policy rate changes to these interest rates is affected by the balance sheet constraint (15) and banking costs (17).

To see how policy rate changes are transmitted, assume that the fraction μ of deposits which lowers the cost reducing effect of reserves equals zero (μ will actually take a value close to zero). Then, the first order condition for bank deposits (19) and the bank's demand for additional reserves (22) can be combined to

$$1/R_t^d = 1 - E_t\varphi_{t,t+1}\left(R_{t+1}^m - 1\right), \tag{30}$$

where the discount factor accounts for the property that the opportunity costs of reserves held by banks in a particular period relate to the current deposit rate R_t^d, whereas their benefit from saving costs of money acquisition becomes effective in the subsequent period. For the particular case where deposits do not provide transaction services, $u_{d,t} = 0$, the households' optimality conditions (3) and (4) reveal that deposits are equivalent to a portfolio of claims with a risk-free payoff, such that $R_t^d = 1/E_t\varphi_{t,t+1}$. For this case, (30) implies that the rate of return on households' saving devices closely relates the expected future policy rate, since $E_t\varphi_{t,t+1}R_{t+1}^m = 1$ and $R_t^d \simeq E_t R_{t+1}^m$. Thus, for this simplified version ($\mu = u_{d,t} = 0$), changes in the monetary policy rate are (almost) completely passed through to the rate that governs the households' consumption and savings decision as in standard models.

For the more general case $u_{d,t} \geq 0$, (30) implies up to a first-order approximation at a steady state $[R - (R_m - 1)] \cdot \widehat{R}_t^d = R^m \cdot E_t\widehat{R}_{t+1}^m - \{R^m - 1\} \cdot \widehat{R}_t$ (where variables with a hat denote percentage deviations from the particular steady state value), which shows that changes in the deposits rate are mainly induced by changes in the expected policy rate (given that the coefficient in the curly brackets is relatively small). Put differently, the net deposit rate $i^d = R^d - 1$ approximately equals the net policy rate $i^d = i^m/(1 + R - R^m) \approx i^m$ in a steady state where the policy rate is close to the risk-free rate R (as for $\eta = 0$), while it will be slightly smaller for plausible values of μ. This effect also tends to reduce the deposit rate compared to the bond rate and the lending rate (see 20 and 21), while the latter additionally differs from the deposit rate by marginal costs of loans. Further note that the real deposit rate can deviate from the risk-free rate due to the marginal utility of deposits that is considered as a short-cut for their transactions services (see 3).

4 Parameter Estimates

The model is estimated with Bayesian techniques. Precisely, we estimate three versions of the model: a version where we do not restrict the parameter of the banking cost function (version NS), a version where the elasticity of banking costs with respect to reserves is restricted to be zero, $\omega = 0$ (version S), and an unrestricted version which is estimated for the subsample 1981Q1 to 2007Q4 (version $NS07$). The latter is estimated to disclose whether the parameter estimates are particularly affected by developments of the recent financial crisis. In this Section, we describe the data and the estimation of parameters. Before, we summarize how we set those that are fixed in the estimation procedure. Notably, the steady states of different model versions are not directly affected by the money supply constraint (see Proposition 1), which facilitates comparisons between model versions and related studies. However, they might differ when the estimations lead to different parameter values that affect the steady state.

4.1 Restricted parameters and priors

Table 1 summarizes the values of the parameters that are not estimated in this paper. Most parameters in the model are shared with comparable studies, while several other parameters are less common or even are specific to the model and are chosen to match observable steady state relations and averages for our sample period .

Starting with the common parameters, we apply a unit value for the intertemporal elasticity of substitution for working time and for deposits, and a degree of habit formation of 0.54.[8] Households devote one third of their time on working, which implies $\nu = 69$, and the household discount factor is set to 0.9901. The capital depreciation rate is set at 0.03, the labor share at 0.7 and investment adjustment costs are set equal to 6.00 (compare Smets and Wouters (2003) for a further discussion on the parameters for euro area). The substitution elasticities ε and ε_n are set to 6, implying steady state mark-ups of 1.2. The degree of price indexation is set 0.3 and the steady state inflation is set to 2 percent to match average inflation in the latter part of the sample and to be broadly in line with the ECB's definition of price stability. The average annual monetary policy interest rate is set to 5 percent per annum. The government spending share is set at 0.18.

Regarding the less common parameter, we set the duration of the long-term console equal to 10 years , implying a decay factor ρ of 0.986. The long-run debt-to-GDP ratio is further set at 70 percent, approximating the values in the euro area prior to the financial crisis. The utility weight of holding deposits φ^d is set at 0.01, which is consistent with a long-run equilibrium ratio

[8] In the setting of the model with a consumption preference shock and habit formation it is not possible to identify the persistence of the preference shock and the habit parameter separately. To avoid the identification problem the habit formation parameter is calibrated to 0.54, compare Smets and Wouters (2003) for the value of the habit formation parameter. The problem of the weakly identified preference shock/habit formation is discussed in Chari, Kehoe, and McGrattan (2007).

Table 1: Values assigned to the calibrated parameters

Parameter	Value	Description
υ	1	Frisch labor supply elasticity
φ^d	1	Intertemporal substitution elasticity of deposits
β	0.9901	Discount factor
ϱ	0.01	Deposit weight in the utility function
ν	69	Labor weight in the utility function
h	0.5466	Habit formation parameter
δ	0.03	Depreciation rate
α	0.7	Labor share
ε	6.00	Substitution elasticity for intermed. goods
ε_n	6.00	Substitution elasticity for working time
ι	0.3	Degree of price indexation
μ	0.02	Minimum reserve ratio
λ	0.1	Fraction of money held outright
$\bar{\kappa}$	1	Money supply parameter
ρ	0.986	Decay factor of government bond
$\bar{\pi}$	2.0 %	Target inflation rate (p.a.)
R_m	6.0	Steady state policy rate (p.a.)
g/y	0.18	Steady state government spending share
pb^T/y	0.7	Steady state debt-to-gdp ratio
d/y	1.2	Steady state deposits-to-gdp ratio

Note: This table shows the values for the calibrated parameters and the steady state ratios.

of deposit-to-gdp of 1.2.[9]. We further set the share of reserves μ that are held for the liquidity management of deposits equal to 0.02 and the share of money supplied via outright purchases to repurchase agreements equal to 0.1, which are broadly consistent with related shares for the sample period. Variations of both parameters were found to be hardly relevant for the estimations (implying nearly identical posterior mode estimates) and for the quantitative results. The means of the stochastic processes, except for the price mark-up shock and the money supply/demand shocks, are set equals to one.

For the prior means, we refer, as far as possible, to estimates in previous studies. Specifically, the prior means for parameter ϕ and ς which govern the degree of price and wage rigidity are set at 0.7. Regarding the fiscal policy rule (26),we follow Reicher (2013) and we set the debt feedback coefficient at 0.06 and coefficient on output at 0.01. The parameter of the interest rate rule (29) are set in a standard way, i.e. with a smoothing factor of 0.7, an inflation coefficient of 1.5 and output coefficient of 0.01. Given that external information on the parameters of the banking cost function

[9] In the model the deposits are narrowly defined as the bank deposits of households, which are then calibrated in line with the Worldbank's estimate of 'bank deposits to GDP'. Note that the ratio of 'bank deposits to GDP ' has been increasing over the sample for the euro area, where we use the value of the ratio towards the end of the sample.

(17) were not available, we conducted estimates for a larger range of priors. For the estimation, for which the results are summarized in Table 2, we set the prior means of the loan elasticities η^{rc} and ratio of the money-to-loan elasticity ω at 0.01 and 2.5.

4.2 Data and shocks

For the estimation, we use quarterly data for the euro area of nine time series from 1981Q1 to 2011Q4. Standard macroeconomic time series are taken from the AWM database. More specifically, we use real GDP growth, real private consumption growth, real investment growth, the private consumption deflator, wage inflation and the monetary policy interest rate (EONIA) to include the core of the workhorse DSGE model.[10] As a measure of money supplied in open market operations, we employ the growth rate of total reserves.[11] We further use the growth rate of loans to the private sector and the associated lending rate. In order to estimate the model we have to consider as many shocks as observable variables in the model. Seven macroeconomic shocks are in common with related studies: A time preference shock (ξ_t), a total factor productivity shock (a_t), an investment technology shock ($\epsilon_{x,t}$)„ a price mark-up shock ($\mu_{m,t}$), a wage mark-up shock ($\epsilon_{w,t}$), a government expenditure shock ($\epsilon_{g,t}$) and a policy rate shock ($\epsilon_{r,t}$). We further consider a shock to the banking cost function (ζ_t) and shocks to money demand, which are either measured by μ_t in the S version or by $\epsilon_{ms,t}$ in version NS version. All shocks are modelled as AR(1) processes, except for shocks to the interest rate rule, which are assumed to be i.i.d. with zero mean.

4.3 Estimation

Employing Bayesian inference methods allows formalizing the use of prior information from earlier studies at both the micro and macro level in estimating the parameters of a possibly complex DSGE model. This seems particularly appealing in situations where the sample period of the data is relatively short, as is the case for the euro area. From a practical perspective, Bayesian inference may also help to alleviate the inherent numerical difficulties associated with solving the highly non-linear estimation problem.

Formally, let $p(\theta|m)$ denote the prior distribution of the parameter vector $\theta \in \Theta$ for some model $m \in M$, and let $L(Y_T|\theta, m)$ denote the likelihood function for the observed data, $Y_T = \{y_t\}_{t=1}^T$, conditional on parameter vector θ and model m. The joint posterior distribution of the parameter vector θ for model m is then obtained by combining the likelihood function for Y_T and the prior distribution of θ,

$$p(\theta|Y_T, m) \propto L(Y_T|\theta, m)p(\theta|m).$$

[10] Since the model does not explain any divergences in trend growth rates of the variables, the growth rates of the observables are centered around zero. For the interest rates we deduct a linear trend.

[11] The time series for total reserves starts in 1999q1 only. We make use of missing data techniques (Giordani, Pitt, and Kohn, 2011). All estimations were conducted using dynare (Adjemian, Bastani, Karamé, Juillard, Maih, Mihoubi, Perendia, Ratto, and Villemot, 2011)

Table 2 shows the posterior mode estimates of the three model versions of the model (NS, S and $NS07$). The estimates of all parameters shared with related studies, specifically the degree of price and wage rigidity, are in line with previous estimates (see Smets and Wouters (2003)). For the estimation of the parameter values for the banking costs function, i.e. the elasticities of banking costs with regard to loans η_{rc} and the ratio of the reserve elasticity to the latter ω, we applied a prior means of one and allowed for a considerably flat distribution. For the unrestricted estimations of the model, we found positive values for both elasticities η_{rc} and $\omega\eta_{rc}$, indicating that banking costs are only slightly affected by loan and money, which are nevertheless non-separable according to these estimates. When the parameter ω is restricted to be zero, the estimates lead to a much larger values – compared to the NS version – for the loan elasticity η_{rc} and the investment adjustment cost parameter γ_X, while the other parameter values are very similar to the estimates for the NS versions. Notably, the standard deviation of the banking cost shock, which will be further analyzed below, is twice as large in the S version than in the NS version. Overall, we find that the unrestricted version is slightly preferred by the data, as indicated by the log data density.

5 Quantitative results

In this Section, we examine quantitative properties of the model, which has been estimated for different versions, for which we either restricted the parameter ω to equal zero implying money to be de facto separable (version S) or we do not restrict the banking cost parameter leading to a version where money is non-separable (version NS). We estimate version S and version NS for the full sample, 1981Q1 to 2011Q4**,** as well as the latter (unrestricted) version for a shorter sample excluding the crisis period 1981Q1 to 2007Q4, which we abbreviate with $NS07$ as we find significant impact of reserves on banking costs. In the first part of this Section, we briefly discuss selected unconditional moments generated by the model. In the second part, we present some impulse response functions, for shocks related to financial intermediation (the full set of impulse response functions are given in the Appendix). In the third part of this Section, we examine the contribution of these shocks to the fluctuations of macroeconomic aggregates and prices.

5.1 Selected moments

Table 3 presents standard deviations of the observable variables and their contemporaneous correlations with output. These unconditional second moments are based on the data and on simulated series of all versions of the model (S, NS, and $NS07$). For all versions, the standard deviations of the simulated series (except for loan growth) tend to overpredict the empirical standard deviations, which is consistent with Smets and Wouters's (2007) result for US data of a similar time period. This is particularly the case for total reserve growth, which is further most volatile in the S version. All model based correlations with output of the NS and the S version accord qualitatively

Table 2: Parameter estimates of the model for the versions NS, S, and $NS07$

Parameter		Prior			Posterior mode		
		Type	*Mean*	*Std*	NS	S	$NS07$
Firms and Households							
Price rigidity	ϕ	B	0.700	0.2000	0.7473	0.7283	0.7496
Wage rigidity	ϱ	B	0.700	0.0200	0.6750	0.6809	0.7027
Investment adjustment cost	γ_X	G	6.000	5.0000	4.3921	8.4739	4.3492
Banks							
Loan elasticity	η^{rc}	G	1	0.7000	0.0178	0.5146	0.0188
Money-to-loan elasticity	ω	G	1	0.7000	0.0982	–	0.0806
Policy							
Interest rate smoothing	ρ_r	B	0.700	0.1000	0.9109	0.8815	0.8969
Inflation coefficient	ρ_π	G	1.500	0.2000	1.8133	1.7422	1.7734
Output coefficient	ρ_y	G	0.010	0.0010	0.0096	0.0097	0.0097
Debt coefficient	τ_b	G	0.060	0.01	0.0641	0.0656	0.0615
Output coefficient	τ_y	G	0.010	0.0050	0.0065	0.0085	0.0054
Shock persistence							
Preference shock	ρ_ξ	B	0.700	0.1000	0.8887	0.8767	0.8437
Technology shock	ρ_a	B	0.700	0.1000	0.9374	0.9392	0.9581
Investment shock	ρ_x	B	0.700	0.1000	0.8707	0.8523	0.8356
Mark-up shock prices	ρ_p	B	0.700	0.1000	0.9754	0.9536	0.9551
Mark-up shock wages	ρ_w	B	0.700	0.1000	0.8411	0.8119	0.6774
Banking cost shock	ρ_ζ	B	0.700	0.1000	0.8186	0.7904	0.8968
Money demand shock	ρ_m	B	0.700	0.1000	0.9553	0.8667	0.9364
Government spending shock	ρ_g	B	0.700	0.1000	0.8990	0.8948	0.8768
Standard deviations							
Preference shock	σ_ξ	G^{-1}	0 0.050	0.5000	0.0246	0.0275	0.0244
Technology shock	σ_a	G^{-1}	0 0.050	0.5000	0.0091	0.0093	0.0083
Interest rate shock	σ_r	G^{-1}	0 0.050	0.5000	0.1170	0.1192	0.1169
Investment shock	σ_x	G^{-1}	0 0.050	0.5000	0.0248	0.0447	0.0243
Price mark-up shock	σ_p	G^{-1}	0 0.050	0.5000	0.0133	0.0129	0.0130
Wages mark-up shock	σ_w	G^{-1}	0 0.050	0.5000	0.0954	0.1055	0.1448
Banking cost shock	σ_ζ	G^{-1}	0 0.050	0.5000	0.0131	0.0271	0.0146
Money demand shock	σ_m	G^{-1}	0 0.050	0.5000	0.0111	0.0208	0.0102
Government spending shock	σ_g	G^{-1}	0 0.050	0.5000	0.0153	0.0153	0.0157
log data density (Laplace appr.)					3463.03	3454.39	2949.40

Note: $\mathcal{B}, \mathcal{G}$ and $\mathcal{G}^{-1}$ correspond to Beta, Gamma and inverse Gamma distributions.

Table 3: Stylized facts

	Standard Deviation (σ_X)				Correlation with output growth ($\rho_{X,Y}$)			
	Data	NS	S	$NS07$	Data	NS	S	$NS07$
output growth	0.55	0.85	0.77	0.74	1.00	1.00	1.00	1.00
consumption growth	0.50	0.72	0.72	0.67	0.87	0.86	0.85	0.84
investment growth	1.51	2.94	2.61	2.53	0.88	0.81	0.73	0.76
total reserves growth	1.77	3.74	4.37	3.47	0.26	0.29	0.03	0.25
loan growth	0.96	1.69	1.59	1.53	0.96	0.55	0.48	0.54
CPI inflation	0.45	0.74	0.72	0.61	-0.31	-0.28	-0.36	-0.28
wage inflation	0.43	0.66	0.62	0.60	0.29	0.00	0.00	0.10
policy rate	1.60	2.97	2.84	2.37	-0.33	-0.15	-0.17	-0.19
lending rate	1.34	2.39	2.92	2.04	-0.42	-0.21	-0.16	-0.23

and most of them also quantitatively to the empirical correlations. The positive correlation of wage growth to gdp growth can however only be reproduced by the $NS07$ version. Notably, the correlation of total reserves to gdp (0.26) can considerably well be reproduced by the NS version (0.29), whereas the S version strongly underpredicts the empirical correlation (0.03). This result is already suggestive for the role of reserves as a relevant component of banking costs. The overall performance (in terms of second moments) of the version $NS07$, which has been estimated with pre-crisis data, is comparable to the version NS, including the correlation of reserves to gdp.

5.2 Impulse responses

In this Section, we examine responses to macroeconomic shocks for the version with non-separable reserves (NS) and the version (S) with de facto separable reserves. All shocks refer to one standard deviation of the estimated processes for the exogenous variables.

Figure 1 shows responses to a positive innovation to the policy rate rule (29), which accords to a shock that is typically considered as the monetary policy shock. The contractionary effects on output, consumption, loans, inflation, and the policy rate are similar in both versions and correspond to monetary policy effects in standard macroeconomic models.[12] Overall, the responses for both versions, S and NS, are very similar, except for the response of total reserves and loans, which decline in a more pronounced way in the NS version. Further, changes in policy rate are passed through almost one-for-to other interest rates one in the S version, whereas the impact responses of the loan rate, the deposit rate, and the bond rate are dampened (by roughly 20%) when reserves affects banking costs (in the NS version).

The Figures 2 and 3 show responses to expansionary shocks to money demand, which either

[12] Given that wages are more rigid than prices, the initail decline in the price level is more pronounced that the decline in the nominal wage, such that the wage rate slightly increases in the first periods.

shift the ratio of reserves to deposits μ_t in version S or enter the money demand condition (22) as disturbances $\epsilon_{md,t}$ in version NS, lead to an increase in reserves. In the NS version (see Figure 2), they further affect real activity, and prices in an expansionary way, though the magnitude of their responses are much smaller than of the magnitude of the reserve response. Notably, shocks to money demand are not also not exactly neutral in the S version (see Figure 3), which seems to be at odds with the concept of separability. The reason is that reserves in fact enter both the balance sheet of banks (15) and the central bank (28), and thereby affect the amount of loans. The impact on real activity and prices is more than thousand times smaller than in the NS model, justifying – together with the variance decomposition – the notion of de facto separability.

A comparison of the impulse responses to other shocks typically considered in the literature, namely, total factor productivity (tfp) shocks, price and wage mark-up shocks, and demand shocks, show a similar pattern (see Online-Appendix). Overall, the responses of the components of aggregate demand, of the inflation rate, and of the wage rate are similar in both versions and (qualitatively) relate to responses in standard models (e.g. Smets and Wouters (2007)). Responses of financial variables mostly share the signs of the deviations from steady state, but can substantially differ with regard to the magnitude. In particular, responses of reserves as well as of the loan rate and the deposit rate are less pronounced when central bank money holdings affect banking costs (NS version).

Figures 4 and 5 show that banking cost shocks increase total reserves in both versions and exert a contractionary effect on real activity and prices. In the S version, shifts in banking costs affect all macroeconomic variables at a similar magnitude. Notably, the responses of real activity and prices are much smaller than the response of reserve in the NS version. In suggests that by increasing their holdings of reserves, banks are able to largely offset the adverse impact of banking cost shocks in the NS version, such that the interest rates, prices, and real activity react only to a small extent.

5.3 Variance decomposition

The variance decomposition of main macroeconomic variables for the versions S and NS are given in the Tables 4 and 5. We first examine the shock contributions to the variance of macroeconomic variables (measured in growth rates) with non-separable money (NS) (see Table 4). Preference shocks are most relevant for the variance of consumption and further contribute strongly to output, reserves, inflation, the policy rate, and the lending rate. Tfp shocks contribute particularly strongly to the variance of reserves and loans, and to a much smaller extent to the components of aggregate demand, which relates to the findings in Smets and Wouters (2003). Shocks to the investment technology contribute most to the investment variance and further explain a large share of the variance of output, inflation, the policy rate, and the lending rate. The largest contributor to

Table 4: Variance decomposition with non-separable money (NS)

Forecast horizon: ∞									
Variable	Shock Contribution								
	ε_ξ	ε_a	ε_x	ε_r	ε_p	ε_w	ε_ζ	ε_m	ε_g
output growth	13.22	5.34	12.03	5.70	17.55	34.20	0.00	10.31	1.64
consumption growth	36.71	4.51	1.60	6.45	16.31	22.95	0.00	11.42	0.05
investment growth	8.69	3.73	39.77	2.38	10.35	30.53	0.00	4.53	0.02
reserves growth	11.85	35.59	2.08	6.81	10.01	7.71	19.06	6.35	0.54
loan growth	8.04	45.85	7.41	2.22	27.06	4.03	0.00	4.58	0.80
CPI inflation	11.03	4.76	9.79	7.80	12.68	27.59	0.00	26.28	0.07
wage inflation	2.62	1.51	3.63	0.68	49.17	40.82	0.00	1.56	0.02
policy rate	12.22	5.42	8.10	4.04	7.01	29.03	0.00	36.14	0.06
lending rate	18.65	8.25	12.34	3.18	10.66	41.34	0.01	5.49	0.08

the variance of output and wage growth is the wage mark-up shock, which is further responsible for large shares of the variance of consumption, inflation, the policy rate, and the lending rate.[13] Compared to wage mark-up shocks, shocks to the price mark-up contribute to a smaller extent to the variance of output, consumption and investment, inflation, and are more relevant for the variance of loans and reserves.

Shocks to the interest rate rule, which are as usual interpreted as money policy shocks, are only responsible for a smaller share of macroeconomic fluctuations. Their contribution to the variance of output, consumption, investment, and inflation is comparable to the contribution of tfp shocks, which differs from the more important role of interest rate shocks in Smets and Wouters (2003), where, however, the policy rate has a direct impact on private sector decision as it is assumed to equal the marginal rate of intertemporal substitution. Banking cost shocks are hardly relevant for the variance of any macroeconomic variable, except of reserves. This accords to the behavior of the impulse response functions, which show that reserves are adjusted to a relatively large amount when banking costs shocks hit the economy (see Figure 4). Shocks to money demand contribute significantly to all variables listed in Table 4, and in particular to output, consumption, inflation, and the policy rate. Notably, they appear to provide a larger contribution to macroeconomic fluctuations than shocks to the monetary policy rate, which is highly suggestive for the view that reserves are non-negligible. Finally, we find that government spending shocks play a minor role for macroeconomic fluctuations (except for the variances of output).

Table 5 presents variance decompositions for the version of the model where the money elas-

[13] This property, i.e. that shocks to the labor market play a major role for macroeconomic fluctuations, is a well-known and critically discussed feature shared with many related studies, and can be mitigated by applying more elaborate specifications of the labor market, like in Smets, Wouters and Gali (2011).

Table 5: Variance decomposition for the version with separable money (S)

Forecast horizon: ∞									
Variable	Shock Contribution								
	ε_ξ	ε_a	ε_x	ε_r	ε_p	ε_w	ε_ζ	ε_m	ε_g
output growth	20.37	6.39	17.59	4.02	16.73	32.87	0.03	0.00	1.99
consumption growth	46.20	5.66	2.77	5.36	15.42	24.51	0.02	0.00	0.06
investment growth	8.29	3.22	59.31	0.64	7.56	20.95	0.01	0.00	0.02
reserves growth	18.33	39.20	4.25	0.81	3.60	8.77	0.14	24.33	0.56
loan growth	10.62	51.55	9.08	0.97	23.91	2.80	0.21	0.00	0.86
CPI inflation	16.88	6.58	14.98	4.83	20.17	36.41	0.08	0.00	0.08
wage inflation	2.61	2.12	6.84	0.34	47.31	40.68	0.09	0.00	0.02
policy rate	20.02	8.06	17.91	4.30	12.97	36.62	0.03	0.00	0.08
lending rate	18.94	7.52	16.83	4.06	13.02	35.20	4.36	0.00	0.08

ticity is restricted to equals zero, $\omega = 0$ (version S). The contribution of non-financial shocks to macroeconomic volatility is comparable to the case of non-separable money demand. The most apparent differences between both versions refer to the financial shocks, namely, shocks to the policy rate, banking costs shocks, and money demand shocks. Compared to the NS version, policy rate shocks play a much smaller role for the variance of investment, total reserve growth, loan growth, and wage growth. Like in the NS version, banking costs shocks are negligible for the volatility of most macroeconomic variables. However, they now significantly affect the variance of the loan rate, while they are irrelevant for the variance of reserves, which is an obvious implication of the property that banking costs are independent of reserves ($\omega = 0$) in version S. Money demand shock only contribute to the fluctuations in reserves, which implies a de facto separability of central bank money in this version. Table 6 further presents variance decompositions for the $NS07$ version. Overall, the results are closely related to the result for the NS version. The main differences to the latter are that wage mark-up shocks contribute less to the variance of all macroeconomic variables except for the wage rate and that shocks to money demand are less important, though they nevertheless contribute more to fluctuations in macroeconomic variables than interest rate shocks. Thus, even in the pre-crisis periods shocks to money demand are non-negligible for macroeconomic dynamics.

Figures 6-7 further show the observed decomposition of output over the sample period for the NS and the S version. Most apparently, they confirm that banking costs shocks are irrelevant in both versions for output growth. Shocks to banks' demand for reserves, namely, μ_t in the S version and $\epsilon_{md,t}$ in the NS version, play a non-negligible role for output fluctuations in the NS version (see Figure 6), while they are entirely irrelevant in the S version (again confirming the de facto separability of reserves). Figures 8 and 9 further present the observe variable decomposition

Table 6: Variance decomposition for the non-separable money version with pre-2007 data ($NS07$)

Forecast horizon: ∞									
Variable	Shock Contribution								
	ε_ξ	ε_a	ε_x	ε_r	ε_p	ε_w	ε_ζ	ε_m	ε_g
output growth	20.83	7.25	14.32	5.98	22.34	19.11	0.00	7.86	2.32
consumption growth	51.67	6.27	1.29	5.96	16.21	10.89	0.00	7.65	0.05
investment growth	8.27	4.28	47.29	2.61	16.13	17.79	0.00	3.61	0.03
reserves growth	13.09	37.10	2.19	6.30	9.68	6.11	20.49	4.38	0.69
loan growth	11.63	43.51	8.04	1.95	27.70	3.20	0.00	2.91	1.05
CPI inflation	11.88	6.49	9.83	7.75	22.55	24.22	0.00	17.19	0.09
wage inflation	2.16	1.90	3.27	0.56	45.83	45.31	0.00	0.95	0.02
policy rate	13.17	10.63	9.76	6.45	16.11	19.55	0.00	24.25	0.09
lending rate	17.51	14.17	13.00	4.46	21.33	25.98	0.02	3.42	0.11

for total reserve growth. Looking through the lens of the NS version, we find that banking costs shocks actually contributed to a large extent to changes in reserves and, in particular, during the last part of the sample. In contrast, banking costs shock have been irrelevant for fluctuations in reserves when the S version is applied (see Figure 9).

6 Conclusion

In this paper, we aim assessing the informational content of banking activities and changes in reserves for macroeconomic dynamics, which have typically been neglected in medium scale macroeconomic models build for estimation purposes. Thus, in contrast to previous studies which examine the role of money for real activity and inflation, we focus on a narrow monetary aggregate (total reserves) rather than broader aggregates (like M1 or M2), which are typically found to be negligible. We estimate different versions of the model applying Bayesian estimation techniques for euro area data, including reserves, bank loans, and interest rates on bank loans (in addition to commonly applied macroeconomic time series). The estimations indicate that banking costs are affected by credit supply and reserve holdings, indicating that (high powered) money matters for real activity and prices. Shocks to the banks' demand for reserves contribute significantly to other macroeconomic variables, while fluctuations in reserves are mainly induced by exogenous shifts of banking costs. Observed variable decompositions further show that these shocks did not contribute to the variance of output. However, stochastic disturbances to money demand did play an important role for the fluctuations in real activity and prices, which has particularly been the case in the last part of the sample including the recent financial crisis.

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A Appendix

A.1 Equilibrium conditions

Definition 1 *The set of equilibrium conditions, which have to be satisfied by the set of sequences* $\{c_t,\ \lambda_t,\ n_t,\ d_t,\ \pi_t,\ w_t,\ \widetilde{w}_t,\ mc_t,\ k_t,\ x_t,\ q_t,\ \eta_t,\ m_t,\ pb_t,\ pb_t^T,\ l_t,\ i_t,\ \tilde{Z}_t,\ y_t,\ s_t,\ R_t^m,\ R_t^L,\ R_t^d,\ R_t^b,\ R_t,\ \varphi_{t,t+1},\ p_t^B,\ b_t^T, g_t, \widetilde{\tau}_t\}_{t=0}^{\infty}$, *is given by*

$$\Xi_{m,t} = -\left(R_t^m - 1\right) - \epsilon_{md,t},\ \textit{if}\ \eta_t = 0 \quad \textit{or} \quad i_t + m_{t-1}\pi_t^{-1} = \mu_t d_t \pi_{t-1}^{-1} \quad \textit{if}\ \eta_t = 0, \tag{31}$$

$$1/E_t R_{t+1}^B = 1/R_t^d - E_t\left(R_{t+1}^m - 1\right)\mu_{t+1}\varphi_{t,t+1}, \tag{32}$$

$$\xi_t u_{c,t} = \lambda_t, \tag{33}$$

$$1/R_t^d = E_t\left[\varphi_{t,t+1}\left(1 + u_{d,t+1}/u_{c,t+1}\right)\right], \tag{34}$$

$$\varphi_{t,t+1} = \left(\beta/\pi_{t+1}\right)\left(\lambda_{t+1}/\lambda_t\right), \tag{35}$$

$$1/R_t = E_t\varphi_{t,t+1}, \tag{36}$$

$$\mu_{p,t} w_t = \left(R_t/R_t^L\right) mc_t \alpha a_t n_t^{\alpha-1} k_{t-1}^{1-\alpha}, \tag{37}$$

$$l_t/R_t^L = w_t n_t, \tag{38}$$

$$w_t = [\varsigma w_{t-1}^{1-\varepsilon_n}\left(\pi_t/\pi_{t-1}\right)^{\varepsilon_n - 1} + (1-\varsigma)\widetilde{w}_t^{1-\varepsilon_n}]^{1/(1-\varepsilon_n)}, \tag{39}$$

$$f_t^1 = f_t^2,\ \textit{where}\ f_t^1 = \widetilde{w}_t \xi_t u_{c,t}\left(w_t/\widetilde{w}_t\right)^{\varepsilon_n} n_t + E_t\beta\varsigma[\left(\pi_{t+1}/\pi_t\right)\left(\widetilde{w}_{t+1}/\widetilde{w}_t\right)]^{\varepsilon_n - 1} f_{t+1}^1, \tag{40}$$

$$\textit{and}\ f_t^2 = \epsilon_{w,t}\nu\xi_t\mu_w\left(w_t/\widetilde{w}_t\right)^{(1+\upsilon)\varepsilon_n} n_t^{(1+\upsilon)} + \beta\varsigma[\left(\pi_{t+1}/\pi_t\right)\left(\widetilde{w}_{t+1}/\widetilde{w}_t\right)]^{(1+\upsilon)\varepsilon_n} f_{t+1}^2,$$

$$\tilde{Z}_t = [\varepsilon/\left(\varepsilon - 1\right)] Z_t^1/Z_t^2,\ \textit{where}\ Z_t^1 = \xi_t c_t^{-\sigma} y_t mc_t + \phi\beta E_t\left(\pi_{t+1}/\pi_t^{\iota}\right)^{\varepsilon} Z_{t+1}^1 \tag{41}$$

$$\textit{and}\ Z_t^2 = \xi_t c_t^{-\sigma} y_t + \phi\beta E_t\left(\pi_{t+1}/\pi_t^{\iota}\right)^{\varepsilon-1} Z_{t+1}^2,$$

$$1 = (1-\phi)(\tilde{Z}_t)^{1-\varepsilon} + \phi\left(\pi_t/\pi_{t-1}^{\iota}\right)^{\varepsilon-1}, \tag{42}$$

$$s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\left(\pi_t/\pi_{t-1}^{\iota}\right)^{\varepsilon}, \tag{43}$$

$$k_t = (1-\delta)k_{t-1} + \epsilon_{x,t}\left(1 - \left(\gamma_X/2\right)\left(\left(x_t/x_{t-1}\right) - 1\right)^2\right) x_t, \tag{44}$$

$$\begin{aligned} 1 &= q_t\epsilon_{x,t}\left(1 - \left(\gamma_X/2\right)\left(\left(x_t/x_{t-1}\right) - 1\right)^2 - \gamma_X\left(\left(x_t/x_{t-1}\right) - 1\right) x_t/x_{t-1}\right) \\ &\quad + \beta E_t\left[\left(\lambda_{t+1}/\lambda_t\right) q_{t+1}\epsilon_{x,t+1}\gamma_X\left(\left(x_{t+1}/x_t\right) - 1\right)\left(x_{t+1}/x_t\right)^2\right], \end{aligned} \tag{45}$$

$$q_t = \beta E_t\left(\lambda_{t+1}/\lambda_t\right)\left[q_{t+1}(1-\delta) + (mc_{t+1}/\mu_{p,t+1})(1-\alpha)a_{t+1}n_{t+1}^{\alpha}k_t^{-\alpha}\right], \tag{46}$$

$$1/R_t^d = 1 - E_t\left(R_{t+1}^m - 1\right)\left(1 - \mu_{t+1}\right)\varphi_{t,t+1} \tag{47}$$

$$1/R_t^L = 1/R_t^d - E_t\varphi_{t,t+1}\mu_{t+1}\left(R_{t+1}^m - 1\right) - \Xi_{l,t}, \tag{48}$$

$$d_t = m_t + E_t p_{t+1}^B b_t + l_t, \tag{49}$$

$$i_t = m_t - m_{t-1}\pi_t^{-1}, \tag{50}$$

$$pb_t = pb_t^T - m_{t-1}, \tag{51}$$

$$pb_t^T = p_t^B b_{t-1}^T, \tag{52}$$

$$R_t^b = \rho p_t^B/(p_{t-1}^B - 1), \tag{53}$$

$$y_t = a_t n_t^{\alpha} k_{t-1}^{1-\alpha}/s_t, \tag{54}$$

$$y_t = c_t + x_t + g_t + \Xi_t, \tag{55}$$

Definition 2 *(where* $u_{c,t} = [c_t - hc_{t-1}]^{-\sigma}$, $u_{d,t} = \varrho d_t^{-\varphi}$, $\Xi_t = \zeta_t[l_t\left(m_{t-1}\pi_t^{-1} - \mu d_{t-1}\pi_t^{-1} + i\right)^{-\omega}]^{\eta_{rc}}$, $\Xi_{l,t} = \eta_{rc}\ \Xi_t/l_t$, $\Xi_{m,t} = -\omega\eta_{rc}\Xi_t\left(m_{t-1}\pi_t^{-1} - \mu d_{t-1}\pi_t^{-1} + i\right)^{-1}$*), as well as the transversality conditions, fiscal and monetary policy satisfying*

$$\widetilde{\tau}_t = g_t - \left(p_t^B - 1\right)\rho^{-1}b_t^T + p_t^B b_{t-1}^T/\pi_t, \tag{56}$$

$$\widetilde{\tau}_t - \widetilde{\tau} = g_t - g + \rho_{\tau b}\cdot\left(p_t^B b_{t-1}^T\pi_t^{-1} - \overline{pb}^T\pi^{-1}\right) + \rho_{\tau y}\cdot(y_t - y), \tag{57}$$

$$R_t^m = \left(R_{t-1}^m\right)^{\rho_R}(R^m)^{1-\rho_R}(\pi_t/\pi)^{\rho_\pi(1-\rho_R)}(y_t/y)^{\rho_y(1-\rho_R)}\exp\varepsilon_{r,t}, \tag{58}$$

and $\overline{pb}^T > 0$, $\overline{\pi} \geq \beta$, *for given initial values* $m_{-1} > 0$, $l_{-1} > 0$, $pb_{-1}^T > 0$, $pb_{-1} > 0$, $k_{-1} > 0$, $x_{-1} > 0$, $\pi_{-1} > 0$, *and* $s_{-1} \geq 1$, *and* $\{\xi_t,\ a_t,\ g_t,\ \mu_{p,t},\ \epsilon_{w,t}\ ,\ \epsilon_{x,t},\ \zeta_t\}_{t=0}^\infty$ *and* $\{\epsilon_{ms,t}\}_{t=0}^\infty$ *or* $\{\epsilon_{md,t}\}_{t=0}^\infty$ *satisfying*

$$\xi_t = \rho_\xi\xi_{t-1} + \left(1 - \rho_\xi\right) + \varepsilon_{\xi,t}, \tag{59}$$

$$a_t = \rho_a a_{t-1} + (1 - \rho_a) + \varepsilon_{a,t}, \tag{60}$$

$$g_t - g = \rho_g\left(g_{t-1} - g\right) + \varepsilon_{g,t}, \tag{61}$$

$$\mu_{p,t} = \rho_p\mu_{p,t-1} + \left(1 - \rho_p\right)\mu_p + \varepsilon_{p,t}, \tag{62}$$

$$\epsilon_{w,t} = \rho_w\epsilon_{w,t} + (1 - \rho_w) + \varepsilon_{w,t}, \tag{63}$$

$$\epsilon_{x,t} = \rho_x\epsilon_{x,t-1} + (1 - \rho_x) + \varepsilon_{x,t}, \tag{64}$$

$$\zeta_t = \rho_\zeta\zeta_{t-1} + \left(1 - \rho_\zeta\right) + \varepsilon_{\zeta,t}, \tag{65}$$

$$\mu_t = \rho_\mu\mu_{t-1} + \left(1 - \rho_\mu\right)\mu + \varepsilon_{\mu,t}\ \text{if}\ \eta_t > 0, \tag{66}$$

$$\text{or}\ \epsilon_{md,t} = \rho_{md}\epsilon_{md,t-1} + \varepsilon_{md,t}\ \text{if}\ \eta_t = 0,$$

and i.i.d. mean zero innovations $\varepsilon_{\xi,t}$, $\varepsilon_{a,t}$, $\varepsilon_{r,t}$, ε_{gt}, $\varepsilon_{p,t}$, $\varepsilon_{w,t}$, $\varepsilon_{x,t}$, $\varepsilon_{\zeta,t}$, *and* $\varepsilon_{ms,t}$ *or* $\varepsilon_{md,t}$.

A.2 Steady state 3

In this Appendix, we examine the deterministic steady state of the economy. Variables without a time index denote the particular steady state values. Consider a competitive equilibrium as given in definition **??**. It can easily be shown that the equilibrium conditions (33)-(55) imply the steady state values $\{c,\ n,\ d,\ \pi,\ w,\ \widetilde{w},\ mc,\ k,\ x,\ q,\ m,\ pb,\ pb^T,\ l,\ i,\ \tilde{Z},\ y,\ s,\ R^L,\ R^d,\ R^b,\ R,\ p^B,\ \eta\}$ to satisfy $\overline{pb}^T = pb^T$, $\overline{\pi} = \pi$,

$$R = \pi/\beta,\ k\delta = x,\ q = 1,\ 1/\beta = (1-\delta) + (mc/\mu_p)(1-\alpha)n^\alpha k^{-\alpha}, \tag{67}$$

$$\nu n^\upsilon = \mu_w^{-1}\widetilde{w}\left((1-h)c\right)^{-\sigma},\ w = \widetilde{w},\ \text{where}\ \ \mu_w = \varepsilon^w/(\varepsilon^w - 1) \tag{68}$$

$$l/R^L = wn,\ mc\alpha n^{\alpha-1}k^{1-\alpha} = \mu_p w\left(R^L/R\right), \tag{69}$$

$$u_d/u_c = (R/R^d) - 1,\ 1/R^d = 1 - (R^m - 1)(1-\mu)(\beta/\pi), \tag{70}$$

$$1/R^L = 1/R^d - (\beta/\pi)\mu(R^m - 1) - \Xi_l, \tag{71}$$

$$d = m + pb + l,\ pb = pb^T - m, \tag{72}$$

$$i = m\left(1 - \pi^{-1}\right), \tag{73}$$

$$\tilde{Z} = \left(\frac{1 - \phi \pi^{(1-\iota)(\varepsilon - 1)}}{1 - \phi} \right)^{1/(1-\varepsilon)}, \; mc = \tilde{Z} \frac{\varepsilon - 1}{\varepsilon} \frac{1 - \phi \beta \pi^{(1-\iota)\varepsilon}}{1 - \phi \beta \pi^{(1-\iota)(\varepsilon - 1)}}, \; s = \frac{1 - \phi}{1 - \phi \pi^{\varepsilon(1-\iota)}} \tilde{Z}^{-\varepsilon}, \quad (74)$$

$$y = n^{\alpha} k^{1-\alpha}/s, \; y = c + x + g + \Xi, \quad (75)$$

$$\Xi_m = -\left(R^m - 1\right) - \epsilon_{md}, \text{ if } \eta = 0 \;\text{ or }\; i + m\pi^{-1} = \mu d \pi^{-1} \;\text{ if } \eta = 0, \quad (76)$$

$$1/R^B = 1/R^d - \left(R^m - 1\right) \mu(\beta/\pi), \; R^B = \rho p^B / \left(p^B - 1\right), \quad (77)$$

where $u_c = [c(1-h)]^{-\sigma}$, $u_d = \varrho d^{-\varphi}$, $u_n = -\nu n^{\upsilon}$, $\Xi = \zeta [l(m\pi^{-1} - \mu d \pi^{-1} + i)^{-\omega}]^{\eta_{rc}}$, $\Xi_l = \eta_{rc} \Xi / l$, and $\Xi_m = -\omega \eta_{rc} \Xi (m\pi^{-1} - \mu d \pi^{-1} + i)^{-1}$. The steady state allocation and the associated prices can be determined with the conditions (67)-(77) for given target values of inflation and real public debt. The debt target implies the steady state transfer to be adjusted in accordance with the consolidated public sector budget constraint (56), while the prevailing monetary policy instruments are chosen in a way that is consistent with the inflation target.

Now consider the set of equilibrium conditions as given in definition 1. Suppose that the central bank sets the inflation target $\overline{\pi}$ and the government set the debt target $\overline{pb}^T$, which satisfy $\overline{\pi} = \pi$ and $\overline{pb}^T = pb^T$. Then, the conditions in (74) directly determine the steady state values $\{\tilde{Z}, s, mc\}$ and the conditions in (67) imply that the steady state values $\{q, R, k/n, x/n\}$ are given by

$$R = \pi/\beta, \; q = 1, \; k/n = \left(\beta \left[mc/\mu_p\right] (1-\alpha) / \left[1 - \beta(1-\delta)\right]\right)^{1/\alpha}, \; x/n = \delta k/n,$$

Using that aggregate production satisfies $y = n \left(k/n\right)^{1-\alpha}/s$ and substituting out y in the resource constraint (see 75), leads to

$$c + g + \Xi = [(k/n)^{1-\alpha} s^{-1} - \delta (k/n)] n, \quad (78)$$

The two conditions in (69) can further be combined to

$$l = \left[mc/\mu_p\right] \alpha n \left(k/n\right)^{1-\alpha} R, \quad (79)$$

Substituting out the real wage rate with $\nu n^{\upsilon} = \mu_w w \left[(1-h)c\right]^{-\sigma}$ (see 68) in $\left[mc/\mu_p\right] \alpha \left(\frac{k}{n}\right)^{1-\alpha} = w \left(R^L/R\right)$ (see 69), gives

$$\nu n^{\upsilon} \mu_w^{-1} \left[(1-h)c\right]^{\sigma} = \left[mc/\mu_p\right] \alpha \left(k/n\right)^{1-\alpha} \left(R/R^L\right) \quad (80)$$

The conditions in (73) and the steady state version of the banking costs function, further imply $\Xi(m,d,l,\pi) = \zeta \left(l \left(m\left(1 + \Lambda^{-1}\right) - \mu d \pi^{-1}\right)^{-\omega}\right)^{\eta_{rc}}$, $\Xi_l(m,d,l,\pi) = \eta_{rc} \Xi / l$, and $\Xi_m(m,d,l,\pi) = -\omega \eta_{rc} \Xi \left(m\left(1 + \Lambda^{-1}\right) - \mu d \pi^{-1}\right)^{-1}$. Equating deposit demand and supply (70), gives

$$1 + u_d/u_c = (\pi/\beta) - \left(R^m - 1\right)(1-\mu), \quad (81)$$

and combining $1/R^d = 1 - (R^m - 1)(1-\mu)(\beta/\pi)$ with (71) leads to $R/R^L = (\pi/\beta)(1-\Xi_l) - (R^m - 1)$. Using latter to eliminate R/R^L in (80), leads to

$$\nu n^{\upsilon} \mu_w^{-1} [(1-h)c]^{\sigma} = \left[mc/\mu_p\right] \alpha (k/n)^{1-\alpha} [(\pi/\beta)(1-\Xi_l) - (R^m - 1)] \tag{82}$$

Further combining the conditions in (72), gives $d = pb^T + l$. Substituting out loans with the latter in (79) and the banking cost terms in (78), (81), and (82), the five steady state values $\{\mathbf{c}, \mathbf{n}, \mathbf{m}, \mathbf{d}, \mathbf{R}^m\}$ can <u>for $\omega > 0$</u> be determined by

$$\mathbf{c} + g = \left[(k/n)^{1-\alpha} s^{-1} - \delta (k/n)\right] \mathbf{n} - \Xi(\mathbf{m}, \mathbf{d}, \mathbf{d} - pb^T, \pi), \tag{83}$$

$$1 + \varrho \mathbf{d}^{-\varphi} [\mathbf{c}(1-h)]^{\sigma} = (\pi/\beta) - (\mathbf{R}^m - 1)(1-\mu), \tag{84}$$

$$\nu \mathbf{n}^{\upsilon} \mu_w^{-1} [(1-h)\mathbf{c}]^{\sigma} = \left[mc/\mu_p\right] \alpha (k/n)^{1-\alpha} \left[(\pi/\beta)(1 - \Xi_l(\mathbf{m}, \mathbf{d}, \mathbf{d} - pb^T, \pi)) - (\mathbf{R}^m - 1)\right], \tag{85}$$

$$\mathbf{d} - pb^T = \left[mc/\mu_p\right] \alpha \mathbf{n} (k/n)^{1-\alpha} (\pi/\beta), \tag{86}$$

$$\Xi_m(\mathbf{m}, \mathbf{d}, \mathbf{d} - pb^T, \pi) = -(\mathbf{R}^m - 1) - \epsilon_{md}, \tag{87}$$

indicating that reserves are non-separable from the allocation. <u>For $\omega = 0$</u>, we can determine the steady state values $\{\mathbf{c}, \mathbf{n}, \mathbf{d}, \mathbf{R}^m\}$ by (84), (86),

$$\mathbf{c} + g = \left[(k/n)^{1-\alpha} s^{-1} - \delta (k/n)\right] \mathbf{n} - \Xi(\mathbf{d}, \mathbf{d} - pb^T, \pi),$$
$$\nu \mathbf{n}^{\upsilon} \mu_w^{-1} [(1-h)\mathbf{c}]^{\sigma} = \left[mc/\mu_p\right] \alpha (k/n)^{1-\alpha} \left[(\pi/\beta)(1 - \Xi_l(\mathbf{d}, \mathbf{d} - pb^T, \pi)) - (\mathbf{R}^m - 1)\right],$$

while reserves m can residually be determined by $m = \mu d \pi^{-1}$, indicating the separability of money. Notably, the minimum requirement ratio μ can affect the steady state allocation (see 84). Finally, (77) determines the bond rate and the bond prices.

A.3 Figures

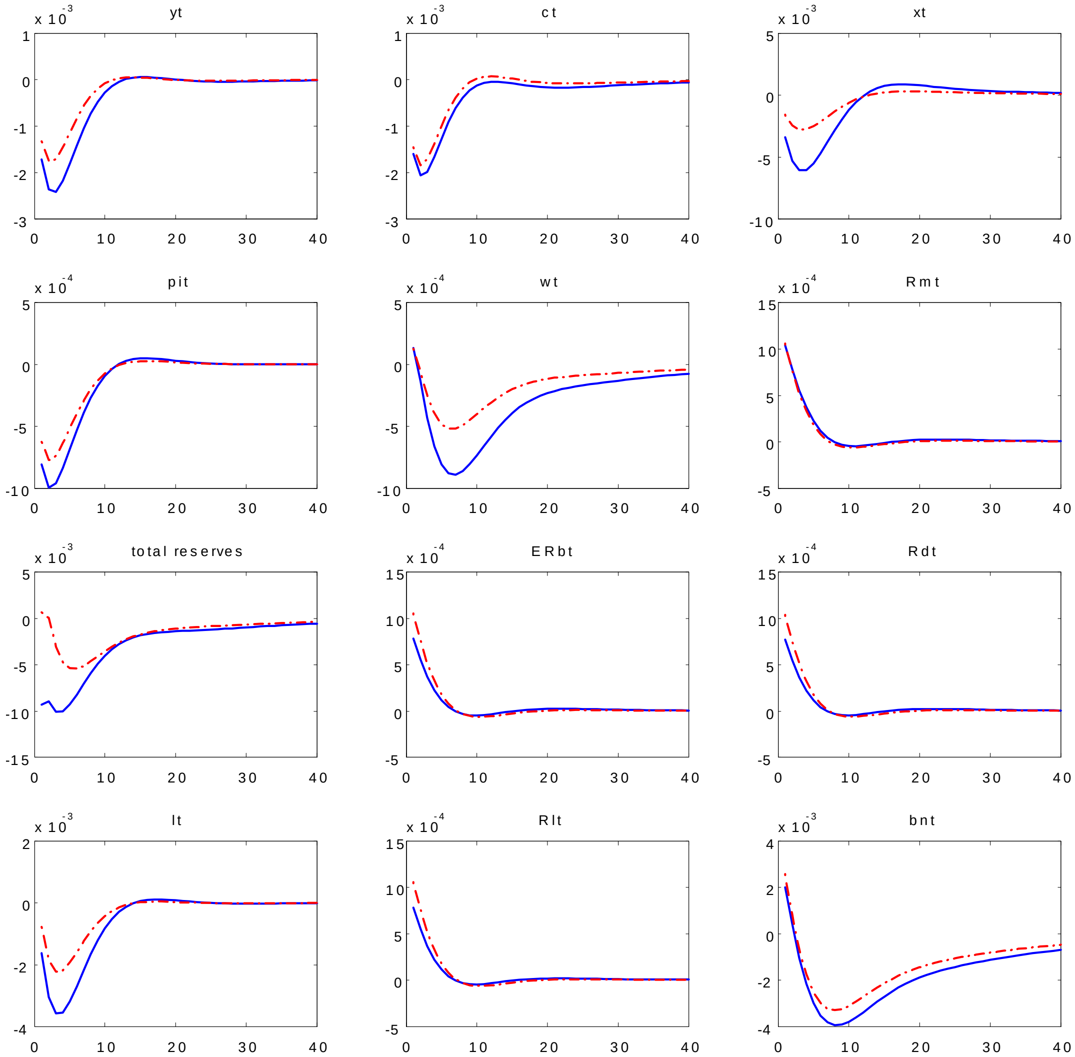

Figure 1: Impulse responses to an interest rate shock (in percent deviations from steady state; NS: blue line, S: red dashed line)

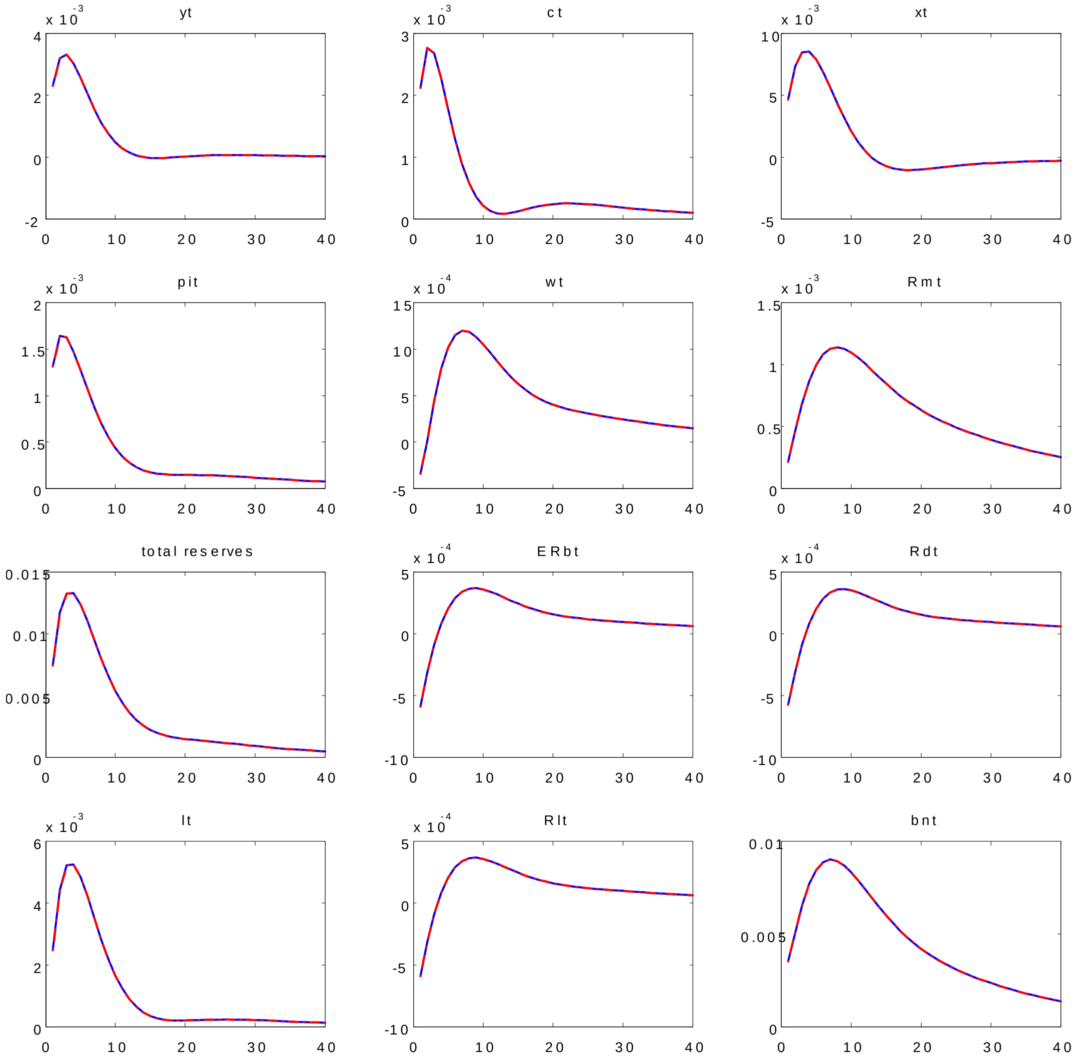

Figure 2: Impulse responses to a money supply/demand shock (in percent deviations from steady state; NS version)

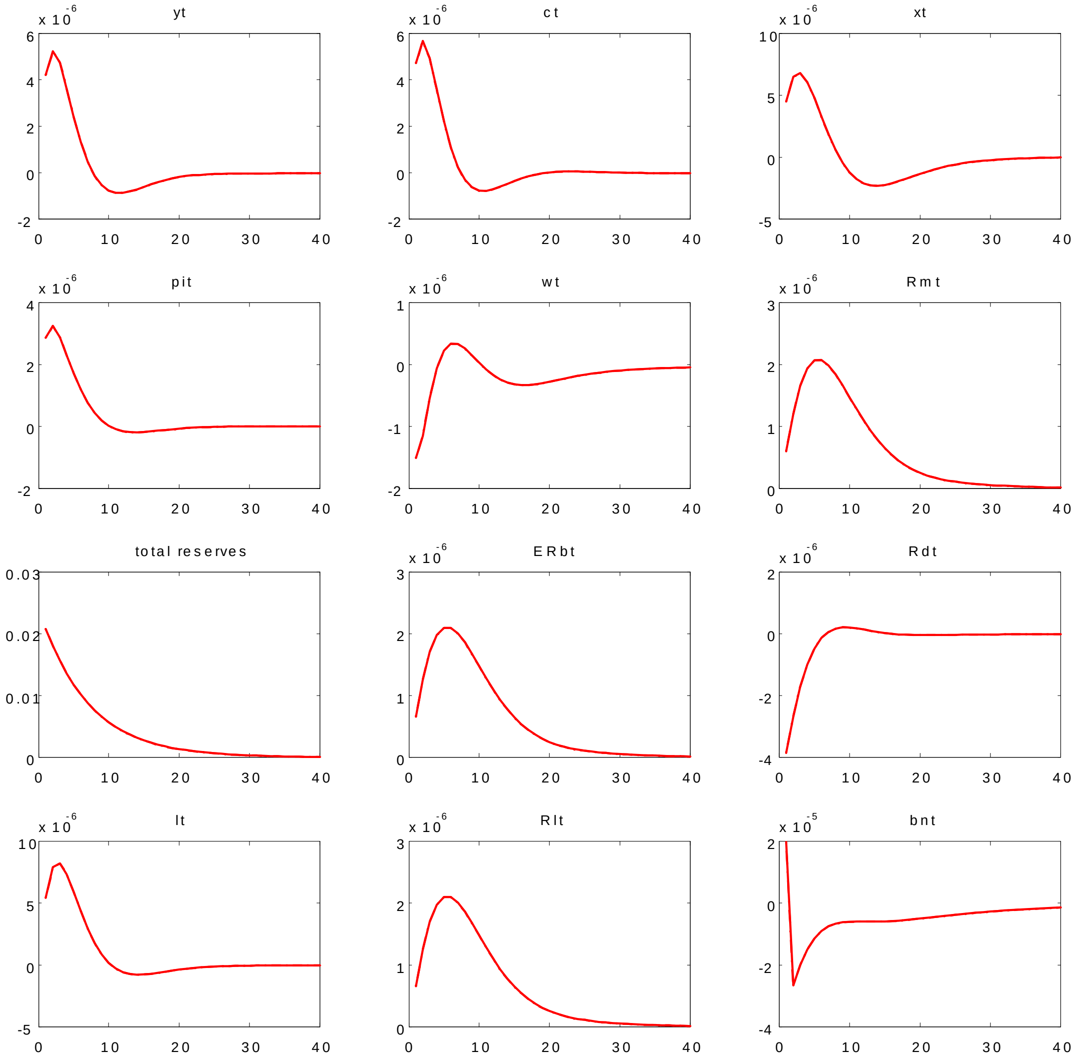

Figure 3: Impulse responses to a money supply/demand shock (in percent deviations from steady state; S version)

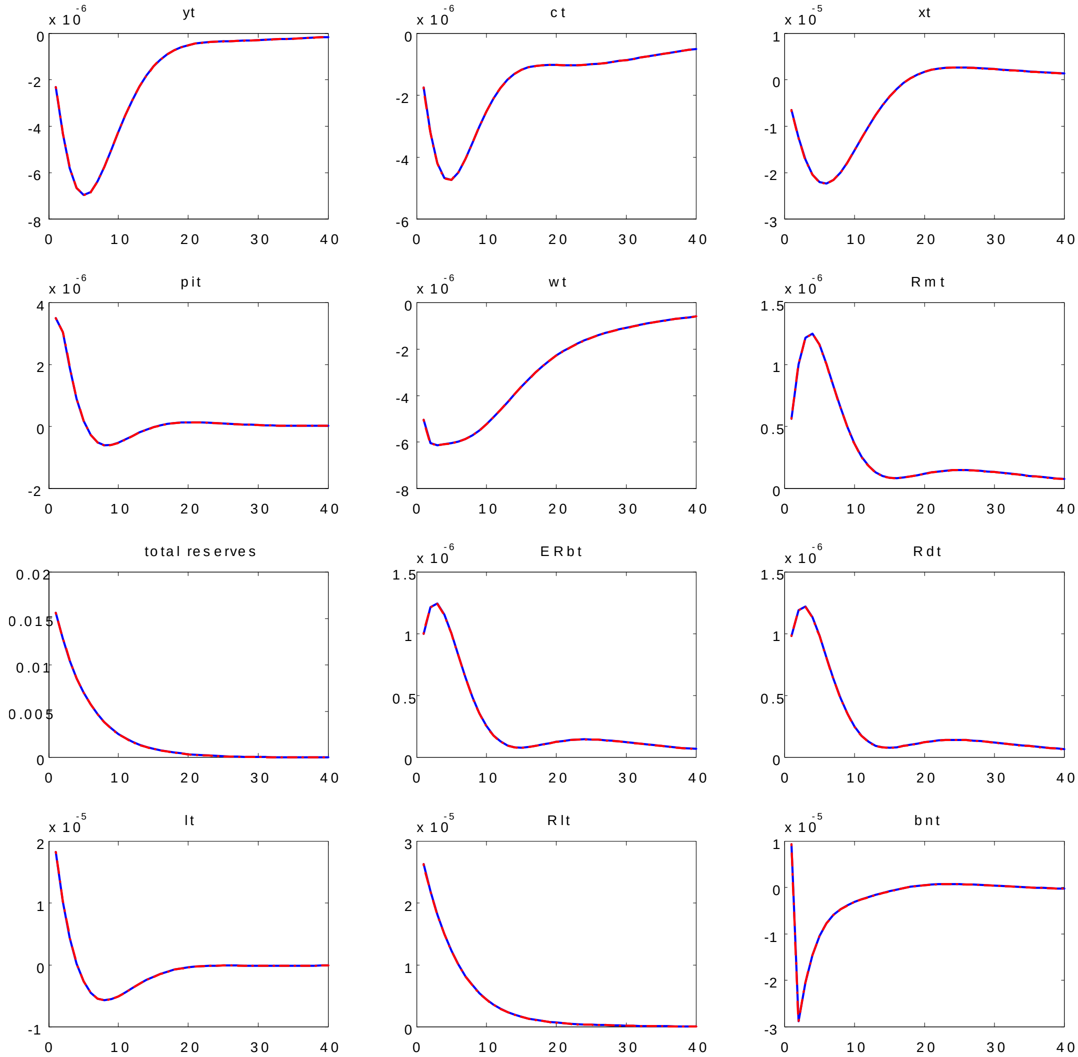

Figure 4: Impulse responses to a banking cost shock (in percent deviations from steady state, NS version)

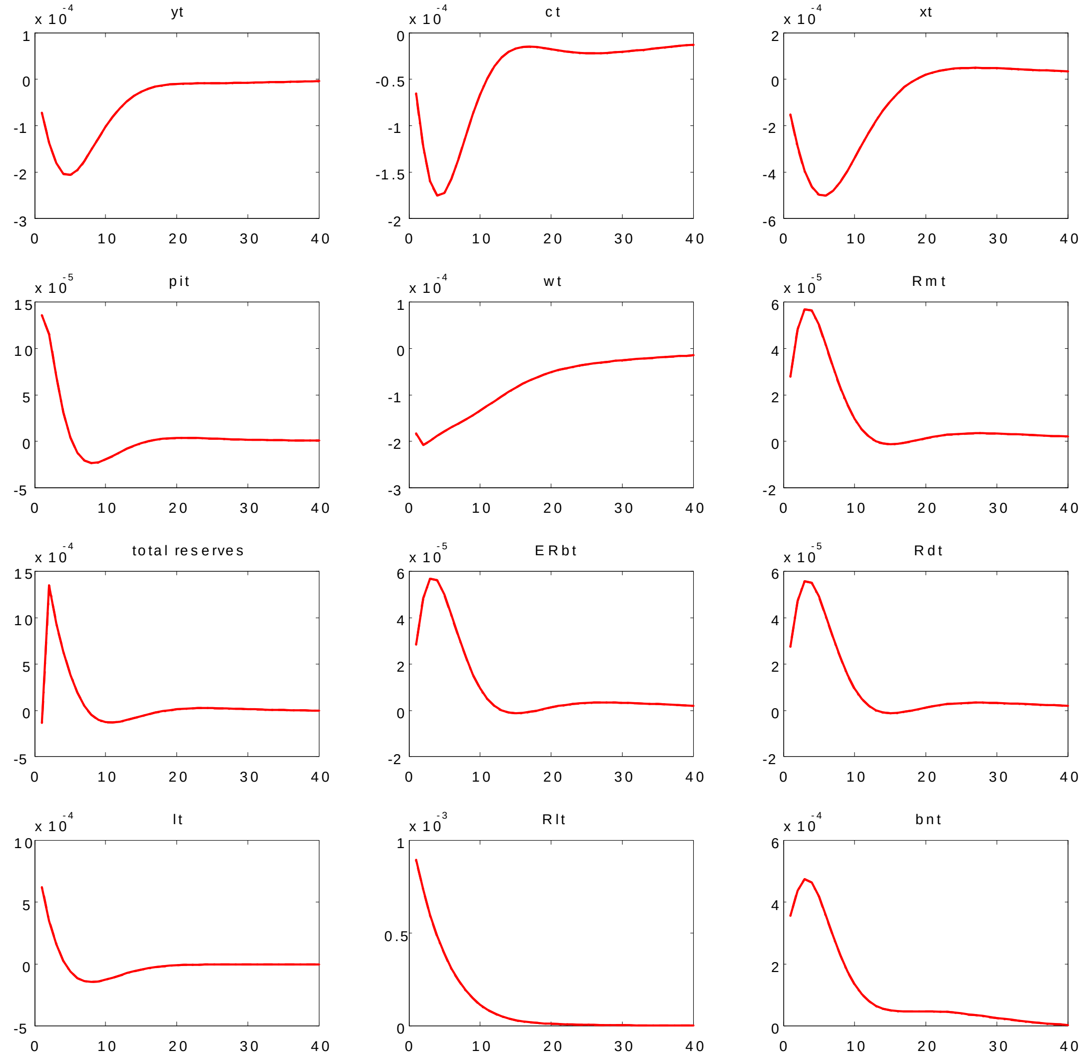

Figure 5: Impulse responses to a banking cost shock (in percent deviations from steady state, S version)

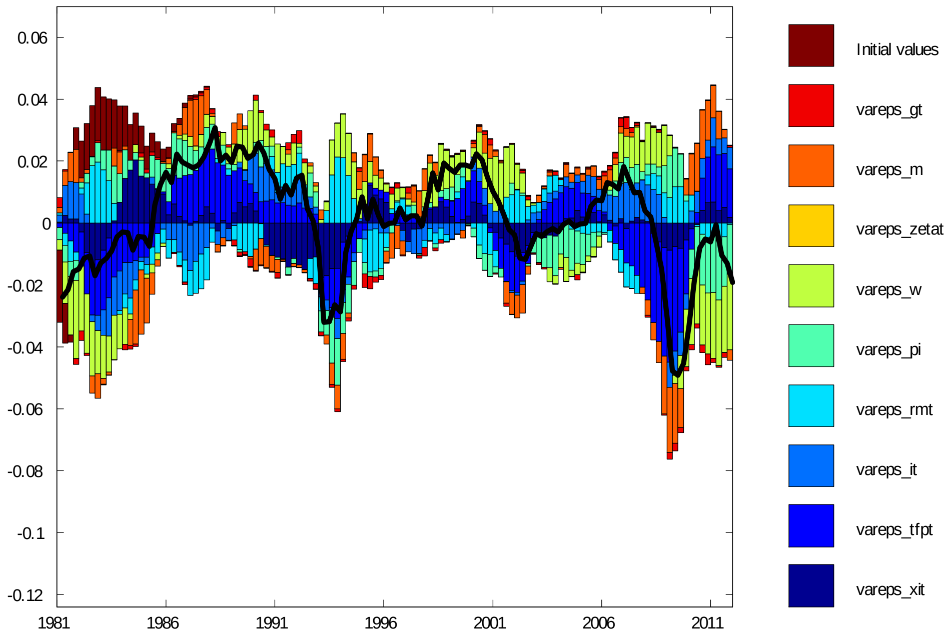

Figure 6: Variance decomposition of output growth for non-separable money

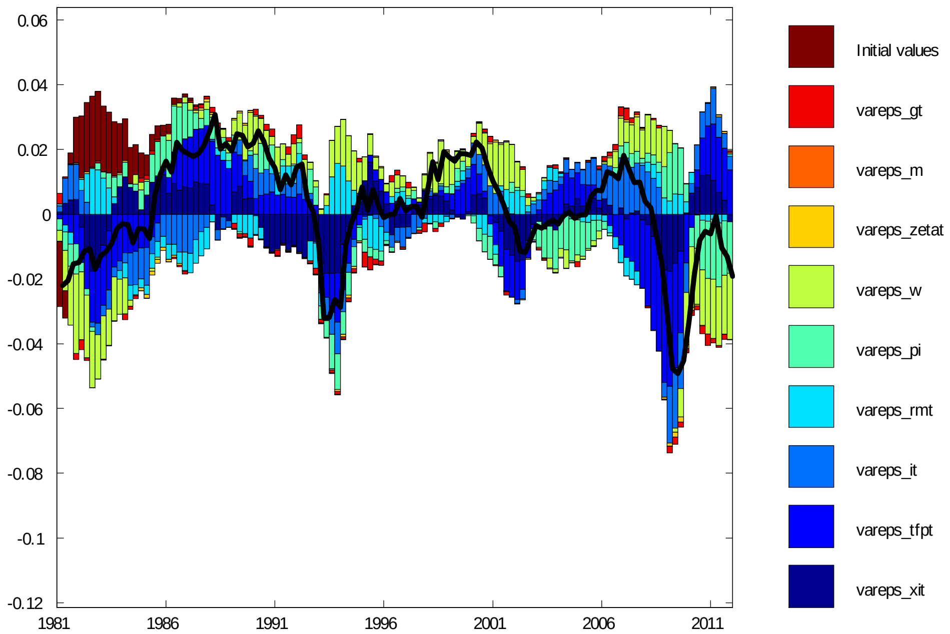

Figure 7: Variance decomposition of output growth for separable money

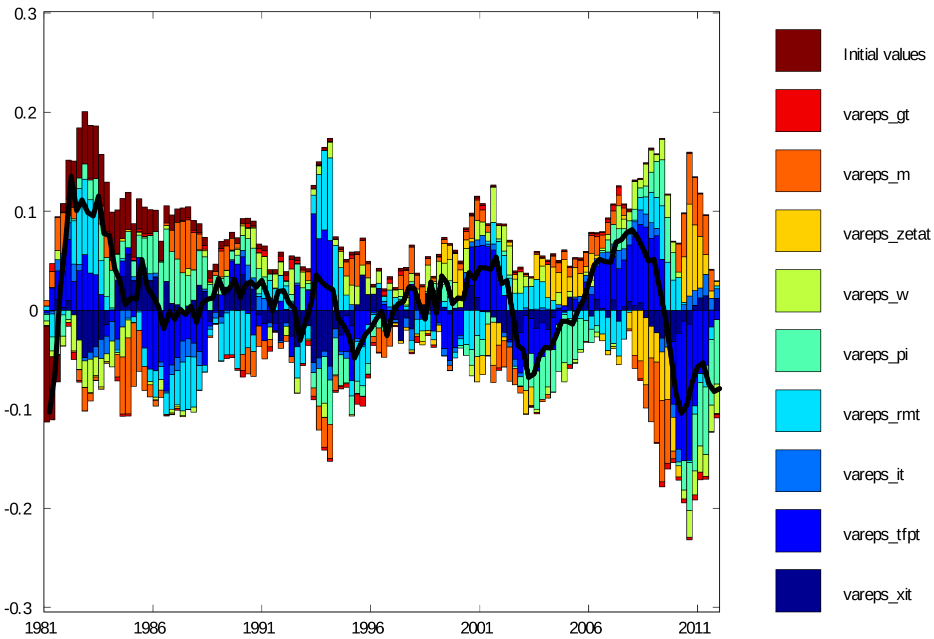

Figure 8: Variance decomposition of total reserve growth for non-separable money

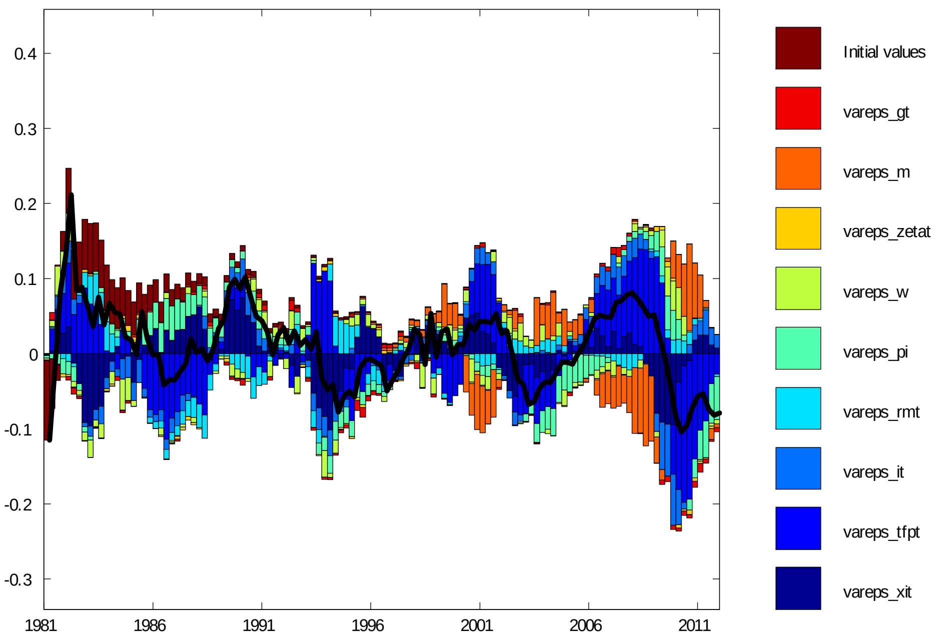

Figure 9: Variance decomposition of total reserve growth for separable money