Revisiting fiscal policy at zero policy rates[1]

Christian Bredemeier
University of Cologne
Falko Juessen
Bergische Universitaet Wuppertal
Andreas Schabert[2]
University of Cologne

This version: June 11, 2014

Abstract

Macroeconomic effects of fiscal policy have regained broad attention in the last few years. Recent studies (e.g. Eggertsson, 2010, Christiano et al., 2011) have shown that fiscal multipliers can be large if nominal interest rates are at the zero lower bound. We consider a framework where the rates of return on non-money market assets differ from the monetary policy rate when the latter is set at the ZLB. Fiscal policy then exerts conventional effects, i.e. higher government spending and increased income tax rates both raise inflation and reduce private consumption.

JEL classification: E32, E63
Keywords: Fiscal multiplier, zero lower bound, consumption Euler equation

[1] Financial support from the Deutsche Forschungsgemeinschaft (SPP 1578) is gratefully acknowledged.
[2] University of Cologne, Center for Macroeconomic Research, Albertus-Magnus-Platz, 50923 Cologne, Germany. Phone: +49 221 470 2483. Email: schabert@wiso.uni-koeln.de.

1 Introduction

The recent financial crisis has led central banks to lower interest rates and governments to spend large amounts of money for fiscal stimulus programs in many countries. At the same time, evidence on the effectiveness of fiscal spending has been mixed. According to the conventional view on fiscal policy (see Baxter and King, 1993), government spending exerts a positive effect on output while private consumption and investment are crowded out. For the case where the central bank sets the short-run nominal interest rate at its zero lower bound (ZLB), Eggertsson (2011), Christiano et al. (2011), and Woodford (2011) argue that fiscal policy can lead to a large crowding-in of private spending. Eggertsson (2011) has further shown that an increase in the income tax rate can potentially stimulate real activity. Recently, some studies have raised doubts about extraordinarily large fiscal policy effects at the ZLB. Specifically, Erceg and Linde (2014) show that the multiplier falls with the size of government expenditures,[3] while Mertens and Ravn (2014) find that government spending might even exert smaller effects than under normal circumstances when the problem of multiple equilibria under Taylor rules at the ZLB is considered.

In this paper, we focus on a different aspect that questions the existence of large fiscal multipliers at the ZLB. A main requirement of the mechanism that leads to a crowding-in in the above cited studies is that all nominal rates of return that are relevant for private agents' intertemporal choices are bound at zero. While the policy rate applies for money market transactions, rates of return on other assets than short-term money market instruments can well be above zero even if the policy rate is at the ZLB (see Ohanian, 2011, for the case of the 2008 US recession). To account for this observation, we examine fiscal policy effects in a framework, where the policy rate and the rate of return on treasury securities are lower than the rates of return on other assets. Specifically, we apply the model by Hoermann and Schabert (2014), who specify the supply of federal funds against eligible assets in open market operations to analyze macroeconomic effects of central bank balance sheet policies. Given that private agents' marginal rate of intertemporal substitution is distinct from the (real) policy rate, i.e. they differ by an endogenous liquidity premium, the ZLB is of minor relevance for the dynamics of private consumption and savings.[4]

Applying simplifying parameter values, we solve the model analytically and show that it unambiguously favors the neoclassical view on fiscal policy,[5] as government spending crowds out private expenditures and income tax cuts stimulate real activity. These results are shown

[3] Braun et al. (2012) further find that the fiscal multiplier at the ZLB is upward biased when models are solved by log-linearization (rather than by non-linear methods).

[4] Notably, the wedge between the marginal rate of intertemporal substitution and the real policy rate is shown by Reynard and Schabert (2013) to be able to explain the systematic behavior of the Euler equation residual as found by Atkeson and Kehoe (2009) and Canzoneri et al. (2007).

[5] Similar effects of fiscal policy shocks can also be found in New Keynesian models when the ZLB is not binding (see Linnemann and Schabert, 2003).

to be (qualitatively) robust for a more realistic version of the model and for a central bank setting the policy rate according to a Taylor rule, holding it constant at some non-negative value, or setting it at its zero lower bound.

Section 2 presents the model. In Section 3, we derive analytical results on fiscal policy effects for a simplified version and we present impulse responses for a calibrated version of the full model. Section 4 concludes.

2 The model

In this Section, we present a variant of the model of Hoermann and Schabert (2014), who study macroeconomic effects of central bank balance sheet policies, including cases where the policy rate is at the ZLB.[6] We add fiscal policy instruments in form of government spending and a flat income tax rate. Throughout the paper, upper case letters denote nominal variables and lower case letters real variables. Though agents are not heterogenous, we introduce indices for individual agents to describe individual choices in a transparent way.

Overview There are infinitely many households, intermediate firms, and retailers, indexed with $i \in [0,1]$, $j \in [0,1]$, and $h \in [0,1]$ respectively, as well as bundlers which are not indexed. A household i enters a period t with holdings of money $M_{i,t}^H \geq 0$, one-period nominally risk-free government bonds $B_{i,t} \geq 0$, and shares of firms $z_{i,t-1} \in [0,1]$ valued at the price V_t. At the beginning of each period, aggregate shocks materialize. First, households and the central bank participate in open market operations, where money is supplied against eligible securities and the price of money, i.e. the policy rate R_t^m, is controlled by the central bank. We assume that only government bonds are eligible,

$$I_{i,t} \leq B_{i,t-1}/R_t^m, \tag{1}$$

where $I_{i,t}$ denotes additional money newly acquired from the central bank. Money is assumed to be the only accepted means of payment for consumption goods $c_{i,t}$. Hence, households face the following cash-in-advance constraint in the goods market which is the second market to open,

$$P_t c_{i,t} \leq I_{i,t} + M_{i,t-1}^H, \tag{2}$$

where P_t denotes the price level. Third, the asset market opens. Before this, households can repurchase treasuries. In the asset market, loans are repaid and households trade money,

[6] In contrast to Hoermann and Schabert (2014), we neglect borrowing by firms and assume that only treasury securities can be pledged as collateral for central bank money, which broadly accords to the US-Federal Reserves' practice before 2007.

equity, and treasuries at the price $1/R_t$ subject to[7]

$$
\begin{aligned}
&(B_{i,t}/R_t) + M_{i,t}^H + (R_t^m - 1)\, I_{i,t} + V_t z_{i,t} + P_t c_{i,t} + P_t \tau_t \\
&\leq B_{i,t-1} + M_{i,t-1}^H + (V_t + P_t \varrho_t)\, z_{i,t-1} + (1 - \tau_t^n) P_t w_t n_{i,t} + P_t \varphi_t,
\end{aligned} \tag{3}
$$

where τ_t denotes a lump-sum tax, ϱ_t dividends from intermediate goods producing firms, τ_t^n a labor income tax, w_t the real wage rate, $n_{i,t}$ working time, and φ_t profits from retailers. The central bank reinvests its payoffs from maturing government bonds in newly issued bonds and leaves aggregate money supply unchanged at this stage, $\int_0^1 M_{i,t}^H di = \int_0^1 (M_{i,t-1}^H + I_{i,t} - M_{i,t}^R) di$, where $M_{i,t}^R$ is money supplied through repos.

Households There is a continuum of infinitely lived households indexed with $i \in [0,1]$. Households have identical asset endowments and identical preferences. Household i maximizes the expected sum of a discounted stream of instantaneous utilities u:

$$
E_0 \sum_{t=0}^{\infty} \beta^t \xi_t u_{i,t}, \tag{4}
$$

where E_0 is the expectation operator conditional on the time 0 information set, and $\beta \in (0,1)$ is the subjective discount factor. The instantaneous utility function is given by $u\,(c_{i,t}, c_{t-1}, n_{i,t}) = [(c_{i,t} - h \cdot c_{t-1})^{1-\sigma} / (1-\sigma)^{-1}] - \theta n_{i,t}^{1+\sigma_n}/(1+\sigma_n)$, where $\sigma \geq 1$, $\sigma_n \geq 0$, $\theta > 0$, and $h \geq 0$, and c_t denotes aggregate consumption. The term ξ_t is a preference shock satisfying $\log \xi_t = \rho_\xi \log \xi_t + \varepsilon_{\xi,t}$ where $\varepsilon_{\xi,t}$ is i.i.d. with zero mean and $\rho_\xi \in [0,1)$. This shock is introduced to replicate the scenario in Eggertsson (2011) that leads to a binding ZLB.

A household i is initially endowed with money $M_{i,-1}^H$, government bonds $B_{i,-1}$, and shares $z_{i,t-1}$. A household faces three constraints in the money market, in the goods market, and in the asset market. In open market operations, it can acquire additional money $I_{i,t}$ up to the amount of government bonds carried over from the previous period B_{t-1} discounted by R_t^m, see (1). When household i leaves the money market, its bond holdings equal $B_{i,t-1} - \Delta B_{i,t}^c$. Households then enter the (final) goods market, where money is assumed to be the only accepted means of payment. Thus goods market expenditures are constrained by money carried over from the previous period plus money acquired from the central bank in current period open market operations, see (2). When household i leaves the goods market, its money stock equals $I_{i,t} + M_{i,t-1}^H - P_t c_{i,t}$. In the asset market, the household receives pay-offs from maturing assets, can buy bonds from the government, and can trade all assets with other households. The term $(R_t^m - 1)\, I_{i,t}$ in (3) measures the costs of money acquired in open market operations: the households receive new cash $I_{i,t}$ in exchange for $R_t^m I_{i,t}$ bonds.

Maximizing the objective (4) subject to the open market constraint (1), the goods market

[7] Further details on the intraperiod flow of funds can be found in Hoermann and Schabert (2014).

constraint (2), the asset market constraints (3) and the borrowing constraints, for given initial values leads to the following first order conditions for working time $n_{i,t}$, consumption $c_{i,t}$, additional money $I_{i,t}$, as well as holdings of contingent claims, government bonds, and money:

$$\xi_t(-u_{n,i,t})/[(1-\tau_t^n)w_t] = \lambda_{i,t}, \tag{5}$$

$$\xi_t u_{c,i,t} = \lambda_{i,t} + \psi_{i,t}, \tag{6}$$

$$R_t^m \left(\lambda_{i,t} + \eta_{i,t}\right) = \lambda_{i,t} + \psi_{i,t}, \tag{7}$$

$$\beta E_t \left[\lambda_{i,t+1} R_{t+1}^q \pi_{t+1}^{-1}\right] = \lambda_{i,t}, \tag{8}$$

$$\beta E_t \left[\left(\lambda_{i,t+1} + \eta_{i,t}\right) \pi_{t+1}^{-1}\right] = \lambda_{i,t}/R_t, \tag{9}$$

$$\beta E_t \left[\left(\lambda_{i,t+1} + \psi_{i,t+1}\right) \pi_{t+1}^{-1}\right] = \lambda_{i,t}, \tag{10}$$

where $u_{n,i,t} = \partial u/\partial n_{i,t}$ and $u_{c,i,t} = \partial u/\partial c_{i,t}$ denote marginal (contemporaneous) utility from labor and consumption, respectively, $R_t^q = (V_t + P_t\varrho_t)/V_{t-1}$ is the nominal rate of return on equity, and $\eta_{i,t}$, $\psi_{i,t}$, and $\lambda_{i,t}$ denote the multipliers on the collateral constraint (1), the goods market constraint (2), and the asset market constraint (3). Finally, the following complementary slackness conditions hold in the household's optimum *i.)* $0 \le b_{i,t-1}\pi_t^{-1} - R_t^m i_{i,t}$, $\eta_{i,t} \ge 0$, $\eta_{i,t}\left(b_{i,t-1}\pi_t^{-1} - R_t^m i_{i,t}\right) = 0$, and *ii.)* $0 \le i_{i,t} + m_{i,t-1}^H \pi_t^{-1} - c_{i,t}$, $\psi_{i,t} \ge 0$, $\psi_{i,t}\left(i_{i,t} + m_{i,t-1}^H \pi_t^{-1} - c_{i,t}\right) = 0$, where $m_{i,t}^H = M_{i,t}^H/P_t$, $b_{i,t} = B_{i,t}/P_t$, and $i_{i,t} = I_{i,t}/P_t$, as well as (3) with equality and the associated transversality conditions.

Conditions (8) and (9) show that the multiplier $\eta_{i,t}$, which measures the liquidity value of treasuries, drives a wedge between the treasury rate R_t and the return on equity R_t^q. The former is closely linked to the policy rate, as households are willing to hold money and treasuries if the rate of return on treasuries compensates for the costs of acquiring new money in the next period (see 7, 9, and 10): $1/R_t = \frac{E_t\left[\left(1/R_{t+1}^m\right)\left(\xi_{t+1}c_{i,t+1}^{-\sigma}/\pi_{t+1}\right)\right]}{E_t\left[\left(\xi_{t+1}c_{i,t+1}^{-\sigma}/\pi_{t+1}\right)\right]}$. Hence, the treasury rate equals the expected future policy rate up to first order which corresponds to observed short-term treasury rates. However, this does not hold for the nominal return on equity which includes a premium which households require to be compensated for the illiquidity of equity.

Firms There are intermediate goods producing firms, which sell their goods to monopolistically competitive retailers. The latter sell a differentiated good to bundlers who assemble final goods using a Dixit-Stiglitz technology. Intermediate goods producing firms indexed with $j \in [0,1]$ are perfectly competitive, owned by the households, and produce an intermediate good $y_{j,t}^m$ with labor $n_{j,t}$ and physical capital $k_{j,t-1}$. Intermediate firm j produces according to the production function

$$y_{j,t}^m = n_{j,t}^{\alpha} k_{j,t-1}^{1-\alpha}, \;\; \alpha \in (0,1), \tag{11}$$

and sells the intermediate good to retailers at the price P_t^m. Physical capital is owned by firms and its accumulation is associated with adjustment costs: $k_{j,t} = (1-\delta)\, k_{j,t-1} + x_{j,t}\Lambda_{j,t}$, where δ is the rate of depreciation, $x_{j,t}$ denotes investment expenditures and investment adjustment costs are $\Lambda\left(x_{j,t}/x_{j,t-1}\right) = 1 - \varsigma\frac{1}{2}\left(x_{j,t}/x_{j,t-1} - 1\right)^2$ with $\Lambda(1) = \Lambda'(1) = 0$ and $\Lambda''(1) = \varsigma > 0$. The problem of a firm j can then be summarized as $\max E_t \sum_{k=0}^{\infty} p_{t,t+k}\varrho_{j,t+k}$, where $p_{t,t+k} = \beta^k \lambda_{i,t+k}/\lambda_{i,t}$ denotes the stochastic discount factor and $\varrho_{j,t}$ real dividends $\varrho_{j,t} = (P_t^m/P_t) n_{j,t}^{\alpha} k_{j,t-1}^{1-\alpha} - w_t n_{j,t} - x_{j,t}$, subject to capital accumulation. The first order conditions for working time, loans, investments, and capital can be summarized by $w_t = mc_t \alpha y_{j,t}^m n_{j,t}^{-1}$,

$$1 = q_t\left[\Lambda_{j,t} + \left(x_{j,t}/x_{j,t-1}\right)\Lambda'_{j,t}\right] - E_t\left[p_{t,t+1}q_{t+1}\left(x_{j,t+1}/x_{j,t}\right)^2 \Lambda'_{j,t+1}\right], \tag{12}$$

$$q_t = E_t\left[p_{t,t+1}\left((1-\alpha) mc_{j,t+1} y_{j,t+1}^m k_{j,t}^{-1} + (1-\delta)\, q_{t+1}\right)\right], \tag{13}$$

where $mc_t = P_t^m/P_t$ denotes real marginal costs and q_t the price of physical capital in terms of the final good. Given that all intermediate goods producing firms face the same prices, they will behave in an identical way.

Monopolistically competitive retailers buy intermediate goods $y_t^m = \int_0^1 y_{j,t}^m dj$ at the common price P_t^m. A retailer $h \in [0,1]$ relabels the intermediate good to $y_{h,t}$ and sells it at the price $P_{h,t}$ to perfectly competitive bundlers, who bundle the goods $y_{h,t}$ to the final consumption good y_t with the technology, $y_t^{\frac{\varepsilon-1}{\varepsilon}} = \int_0^1 y_{h,t}^{\frac{\varepsilon-1}{\varepsilon}} dh$, where $\varepsilon > 1$. The cost minimizing demand for $y_{h,t}$ is therefore given by $y_{h,t} = \left(P_{h,t}/P_t\right)^{-\varepsilon} y_t$. Retailers set their prices to maximize profits, where we consider a nominal rigidity in form of staggered price setting. Each period, a measure $1-\phi$ of randomly selected retailers may reset their prices independently of the time elapsed since the last price setting, while a fraction $\phi \in [0,1)$ of retailers do not adjust their prices. The fraction $1-\phi$ of retailers set their prices to maximize the expected sum of discounted future profits, $\max_{\overline{P}_{h,t}} E_t \sum_{s=0}^{\infty} \phi^s p_{t,t+k}(\overline{P}_{h,t} y_{h,t+s} - P_{t+s} mc_{t+s} y_{h,t+s})$, s.t. $y_{h,t+s} = \overline{P} P_{h,t}{}^{-\epsilon} P_{t+s}^{\epsilon} y_{t+s}$. The first order condition for their price $\overline{P}_{h,t}$ is given by $Z_t = \frac{\varepsilon}{\varepsilon-1} Z_{1,t}/Z_{2,t}$, where $Z_t = \overline{P}_{h,t}/P_t$, $Z_{1,t} = c_t^{-\sigma} y_t mc_t + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{1,t+1}$ and $Z_{1,t} = c_t^{-\sigma} y_t + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{2,t+1}$. With perfectly competitive bundlers and the homogeneous bundling technology, the price index P_t for the final consumption good satisfies $P_t^{1-\varepsilon} = \int_0^1 P_{h,t}^{1-\varepsilon} dh$. Using the demand constraint, we obtain $1 = (1-\phi)\, Z_t{}^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}$.

Aggregate intermediate output is then given by $y_t^m = n_t^{\alpha} k_{t-1}^{1-\alpha}$, where $n_t = \int_0^1 n_{j,t} dj$, while price dispersion across retailers affects aggregate final output. Specifically, the market clearing condition in the intermediate goods market, $y_t^m = \int_0^1 y_{h,t} dh$, gives $n_t^{\alpha} k_{t-1}^{1-\alpha} = \int_0^1 \left(P_{h,t}/P_t\right)^{-\varepsilon} y_t dh \Leftrightarrow y_t = n_t^{\alpha} k_{t-1}^{1-\alpha}/s_t$, where $s_t \equiv \int_0^1 \left(P_{h,t}/P_t\right)^{-\varepsilon} dh$ and $s_t = (1-\phi) Z_t{}^{-\varepsilon} + \phi s_{t-1}\pi_t^{\varepsilon}$ given s_{-1}.

Public sector The public sector consists of a government and a central bank. The government issues one-period bonds whose total supply is denoted by B^T and which are held by households (amount B) and by the central bank (amount B^C), $B_t^T = B_t + B_t^C$. For simplicity, we assume that the supply of government bonds is exogenously determined and is issued at a constant growth rate Γ satisfying

$$B_t^T = \Gamma B_{t-1}^T, \tag{14}$$

where $\Gamma > \beta$. It should be noted that we do not aim at modelling the evolution of total public debt by (14). Of course, public debt also consists of government bonds with longer maturity that might grow with a rate different from Γ, which will not be modelled here to keep the exposition simple. Hence, (14) can be viewed as a supply of a particular asset that the central bank declares eligible rather than a characterization of total public debt. In order to avoid any further effects of fiscal policy, we assume that the government can raise or transfer revenues in a non-distortionary way, $P_t\tau_t$. In addition, we consider flat-rate income taxes τ^n which are, like government expenditures g, exogenously determined,

$$\tau_t^n = \rho_\tau \tau_{t-1}^n + (1-\rho_\tau)\tau^n + \varepsilon_{\tau,t},$$
$$g_t = \rho_g g_{t-1} + (1-\rho_g)g + \varepsilon_{g,t},$$

where $\rho_g, \rho_\tau < 1$, $g, \tau^n > 0$ and the innovations $\varepsilon_{g,t}$ and $\varepsilon_{\tau,t}$ have zero mean and are i.i.d..[8] Accounting for the transfers $P_t\tau_t^m$ from the central bank, the simplified government budget is balanced by

$$\left(B_t^T/R_t\right) + \tau_t^n P_t w_t n_{i,t} + P_t \tau_t^m = P_t g_t + B_{t-1}^T + P_t \tau_t.$$

The central bank supplies money in exchange for treasuries either outright, M_t^H, or under repos M_t^R. At the beginning of each period, the central bank's stock of treasuries equals B_{t-1}^C and the stock of outstanding money equals M_{t-1}^H. It then receives an amount ΔB_t^C of treasuries in exchange for newly supplied money $I_t = M_t^H - M_{t-1}^H + M_t^R$, and, after repurchase agreements are settled, its holdings of treasuries and the amount of outstanding money reduce by B_t^R and by $M_t^R = B_t^R$, respectively. Before the asset market opens, where the central bank can invest in new T-bills B_t^C, it holds an amount equal to $\Delta B_t^C + B_{t-1}^C - B_t^R$. Its budget constraint is thus given by $\left(B_t^C/R_t\right) + P_t\tau_t^m = \Delta B_t^C + B_{t-1}^C - B_t^R + M_t^H - M_{t-1}^H - \left(I_t - M_t^R\right)$ Substituting out I_t, B_t^R, and ΔB_t^C using $\Delta B_t^C = R_t^m I_t$, it can be simplified to $\left(B_t^C/R_t\right) - B_{t-1}^C = R_t^m \left(M_t^H - M_{t-1}^H\right) + \left(R_t^m - 1\right) M_t^R - P_t\tau_t^m$. Interest earnings are transferred to the government, $P_t\tau_t^m = B_t^C\left(1 - 1/R_t\right) + \left(R_t^m - 1\right)\left(M_t^H - M_{t-1}^H + M_t^R\right)$, and that maturing

[8] We abstract from an explicitly accounting for the bounds $g_t \geq 0$ and $\tau_t^n < 1$ and, in the analytical evaluation, assume that they are satifsfied. In the following numerical analysis, we choose the variances of $\varepsilon_{\tau,t}$ and $\varepsilon_{g,t}$ such that $g_t < 0$ and $\tau_t^n > 1$ are extremely unlikely.

assets are rolled over, such that holdings of treasuries evolve according to $B_t^C - B_{t-1}^C = M_t^H - M_{t-1}^H$. Further restricting initial values to $B_{-1}^C = M_{-1}^H$ leads to the central bank balance sheet constraint

$$B_t^C = M_t^H. \tag{15}$$

Regarding the implementation of monetary policy, we assume that the central bank sets the policy rate according to a simple instrument rule,

$$R_t^m = \left(R_{t-1}^m\right)^{\rho_R} (R^m)^{1-\rho_R} (\pi_t/\pi)^{\rho(1-\rho_R)} (y_t/\widetilde{y}_t)^{\rho_y(1-\rho_R)} \tag{16}$$

where $\widetilde{y}_t$ is the efficient level of output and $\rho \geq 0$, $\rho_y \geq 0$, $\rho_R \geq 0$ and $R^m \geq 1$, which includes pegs at a value $R^m \geq 1$. The target inflation rate, $\pi > \beta$, is controlled by the central bank and is here assumed to equal the growth rate of treasuries for simplicity. In Hoermann and Schabert (2014) it is shown how the central bank can implement its inflation target even if $\pi \neq \Gamma$.[9] Finally, the central bank decides on how much money is traded in form of outright sales/purchases and how much in form of repurchase agreements, $\Omega \geq 0 : M_t^R = \Omega \cdot M_t^H$.

Equilibrium properties Given that all households (firms) behave in an identical way, we will omit the indices i, j, and h in the subsequent analysis. The main difference to a standard New Keynesian model is the existence of the money supply constraint (1), which restricts households' access to money by their holdings of government bonds. The model reduces to a conventional model (see Eggertsson, 2011, or Christiano et al., 2011) if the collateral constraint, $M_t^H - M_{t-1}^H + M_t^R \leq B_{t-1}/R_t^m$, is slack, i.e. if the multiplier η_t equals zero. In this case, there is no liquidity premium on eligible securities, such that the expected equity return equals the treasury rate up to first order (see 7 and 9). Throughout the subsequent analysis, we are particularly interested in the case where the money supply constraint (1) is binding. To see when this is the case, eliminate $\lambda_{i,t}$ and $\psi_{i,t}$ by $\xi_t u_{c,i,t} = \lambda_{i,t} + \psi_{i,t}$ in (7) and in (10), which leads to $\frac{\eta_{i,t}}{\xi_t u_{c,i,t}} = \frac{1}{R_t^m} - \beta E_t \frac{\xi_{t+1} u_{c,i,t+1}}{\xi_t u_{c,i,t} \pi_{t+1}}$. In equilibrium, it can be written as

$$\eta_t = \xi_t u_{c,t}[(1/R_t^m) - (1/R_t^{Euler})] \geq 0, \tag{17}$$

where R_t^{Euler} is the consumption Euler equation rate defined as $1/R_t^{Euler} = \beta E_t \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t} \pi_{t+1}}$. As it is well-known, the nominal Euler equation rate measures the equilibrium valuation of money. Agents are willing to spend $R_t^{Euler} - 1$ to transform one unit of an illiquid asset, i.e. an asset that is not accepted as a means of payment today and delivers one unit of money tomorrow, into one unit of money today. Hence, if the central bank supplies money at a lower price, $R_t^m < R_t^{Euler}$, households earn a positive rent and are willing to acquire the maximum amount of money. Given that this amount is restricted by holdings of eligible

[9] To implement an inflation target π for $\pi \neq \Gamma$, variable sets of eligible securities have to be taken into account.

assets, the money supply constraint (1) will be binding, indicating a positive liquidity value of treasuries, $\eta_t > 0$.[10] A definition of the rational expectations equilibrium can be found in Appendix A.

3 Fiscal policy effects

In this Section, we examine the effects of shocks to the fiscal policy instruments (g_t and τ_t). Given that the main novel contribution of this paper is to examine fiscal policy in a model where the (real) monetary policy rate differs from the marginal rate of intertemporal substitution, we focus on the case where the multiplier on the money market constraint (1) is strictly positive, $\eta_t > 0$.[11] In the first part of this Section, we examine fiscal policy shocks in a simplified version of the model, which can be solved analytically. In the second part, we apply a more realistic version of the model and present impulse response functions of a numerical solution.

3.1 Analytical results

To facilitate the derivation of analytical results, we simplify the model by applying the parameter values $\alpha = 1$, $h = 0$, and $\pi = \Gamma = 1$, which implies that there is no capital accumulation, no habit formation, and long-run price stability. We further simplify monetary policy by restricting the policy rate to be set according to $\rho_y = \rho_R = 0$. A definition of the rational expectations equilibrium in this model version can be found in Appendix B. Under the applied parameter restrictions, we log-linearize the model at a steady state with a binding money market constraint and derive a (sufficient) condition for equilibrium determinacy, i.e. for the existence and the uniqueness of locally convergent equilibrium sequences. It implies that, under a binding money market constraint (1), an active monetary policy ($\rho > 1$) is neither necessary nor relevant for equilibrium determinacy in this model and that the central bank can peg the policy rate ($\rho = 0$) without inducing indeterminacy (see also Hoermann and Schabert, 2014). This property is mainly due to the existence of a non-explosive stock of nominal assets that serves as collateral for money, which provides a nominal anchor for monetary policy similar to a fixed growth rate of money. Then, a passive policy does not allow for multiple equilibria, given that, for $\eta_t > 0$, the central bank just controls the price of money rather than the marginal rate of intertemporal substitution.

Proposition 1 *For $h = \rho_y = \rho_R = 0$, and $\alpha = \pi = \Gamma = 1$, the equilibrium under binding money market constraint (1) is locally stable and unique if but not only if*

$$\rho < \frac{1+\beta}{\chi\sigma} + \frac{1-\sigma}{\sigma}. \tag{18}$$

[10] Notably, a binding collateral constraint (1), which relies on a positive valuation of liquidity, implies that the cash constraint (2) is binding as well, $\psi_t > 0$.

[11] A version where the (real) policy rate is – as usual – assumed to be identical to the marginal rate of intertemporal substitution is included for comparisons in the Figures 1 and 2.

Proof. See appendix B. ■

It should be noted that the determinacy condition (18) is far from being restrictive for a broad set of reasonable parameter values. Consider for example the parameter values $\beta = 0.993$, $\sigma = 1$ and $\phi = 0.8$ (see Section 3.2). Then, the upper bound $\frac{1+\beta}{\chi\sigma} + \frac{1-\sigma}{\sigma}$ equals 38.8, which is far from any value estimated for the inflation feedback ρ. Given that a peg is also associated with equilibrium determinacy, the model can easily be used to assess the impact of fiscal policy when monetary policy holds the policy rate fixed at the zero lower bound (a peg with $R^m = 1$). Specifically, we show that the effects of fiscal policy shocks, i.e. an increase in government spending and an increase in the tax rate, lead to a decline in private consumption and to an increase in inflation regardless of the level of the mean policy rate and of the inflation feedback ρ. For this, we further simplify the model by assuming $\Omega \to \infty$, which implies that money is only supplied temporarily (under repos).

Proposition 2 *For $h = \rho_y = \rho_R = 0$, $\alpha = \pi = \Gamma = 1$, $\Omega \to \infty$, and (18), an unexpected increase in government consumption and an unexpected rise in the income tax rate both lead to lower private consumption and higher inflation on impact.*

Proof. See appendix B ■

As summarized in Proposition 2, the model predicts that fiscal policy leads to conventional effects regardless whether the policy rate is fixed at some value $R^m \geq 1$ or increased with inflation, if (18) is satisfied. Notably, these results also apply for the case where the policy rate is pegged at the ZLB. Specifically, an increase in government spending crowds-out private consumption such that the fiscal multiplier is necessarily smaller than one. Likewise, an increase in the labor income tax rate increases inflation and reduces private consumption. This result substantially differs from the results derived in Eggertsson (2011), where not only the monetary policy rate is fixed at the ZLB, but all nominal rates of return behave in an identical way. To demonstrate the robustness of these results, we apply more realistic parameter values in the subsequent Section.

3.2 A calibrated version

We apply standard parameter values (which accord to an interpretation of a period as a quarter) as far as possible. We follow Christiano et al. (2005) and apply their values for their non-estimated parameters and set the inverses of the elasticities of intertemporal substitution at $\sigma = 1$ and $\sigma_n = 1$, the labor income share at $\alpha = 2/3$, and the depreciation rate at $\delta = 0.025$. For the fraction of non-optimally price adjusting firms ϕ, and the elasticity of substitution ϵ we chose the values $\phi = 0.8$ and $\epsilon = 6$, and the utility parameter θ is chosen to lead to a steady state working time of $n = 1/3$. For the investment adjustment cost parameter ς, we apply a value of $\varsigma = 0.065$ that accords to estimates based on disaggregate data (see Groth and Khan, 2010) . For the policy rate, we set the average value equal to the sample

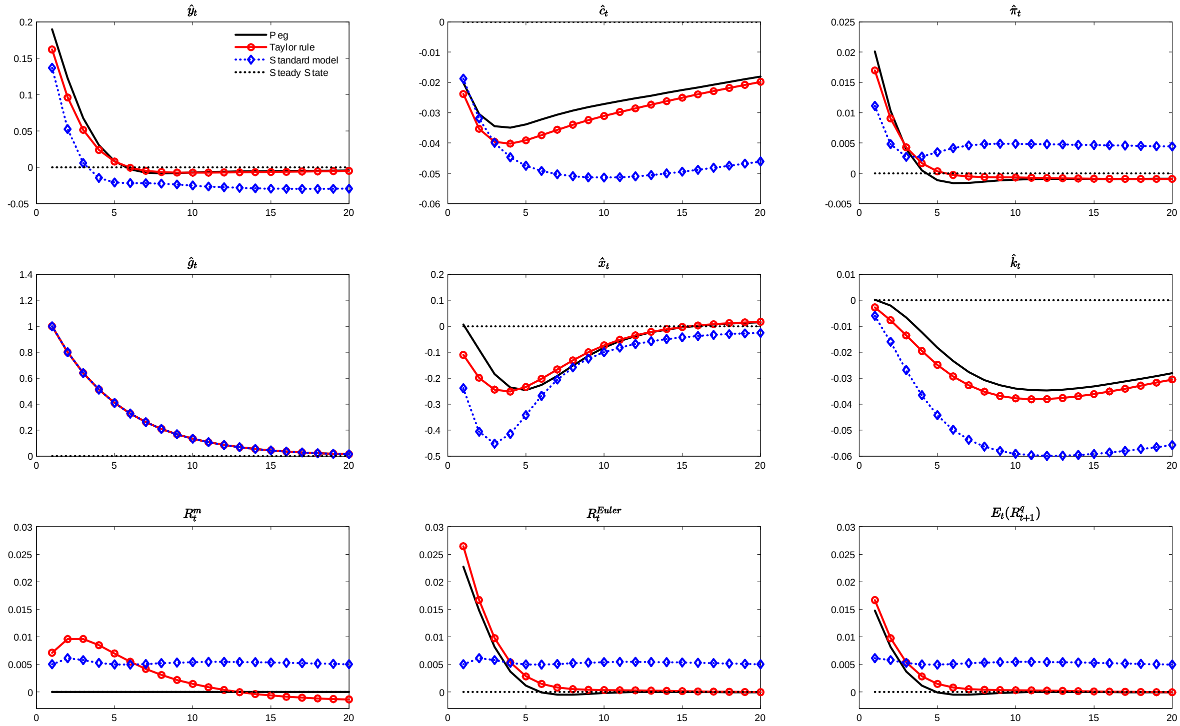

Figure 1: Responses to a positive government spending shock (in %)

mean of the Federal Funds rate for the sample 1966-2007, $R^m = 1.065^{1/4}$. The inflation target is set equal to the mean CPI inflation rate of the same sample period, $\pi = 1.046^{1/4}$.[12] The discount factor β is set to an intermediate value, $\beta = 0.993$, which implies that the steady state spread between the Euler rate and the treasury rate equals 0.0025 (or 100 basis points for annualized rates), which accords to the (AAA) corporate bond yield spread in Krishnamurthy and Vissing-Jorgensen (2012).[13] The growth rate Γ of T-bills (see 14) is set equal to the inflation target, which accords to empirical growth rate of the total stock of T-bills for 1966-2007. Following Reynard and Schabert (2013), we set the policy parameter Ω equal to 25. Finally, the policy rate is either pegged at its steady state value or set according to the interest rate rule with coefficients that are set at standard values $\rho = 1.5$, $\rho_y = 0.25^{1/4}$, and $\rho_R = 0.8$, and the mean tax rate τ^n and the mean government share g/y are set to 0.2 (see Christiano et al., 2011).

Figure 1 shows impulse responses to an autocorrelated ($\rho_g = 0.8$) increase in government spending by one percent. The black solid line refers to a peg ($\rho = 0$), the red line with circles refers to a policy rate set according to a Taylor rule, and the blue dotted line with diamonds refers to a (standard) version of the model with a Taylor rule and where the (real) policy rate

[12] Data for the Federal Funds rate and the inflation rate are taken from FRED database.

[13] Applying a second-order approximation of the model, this spread implies an equity premium $E_0(R_t^q - R_t)$ of 2.32% per annum.

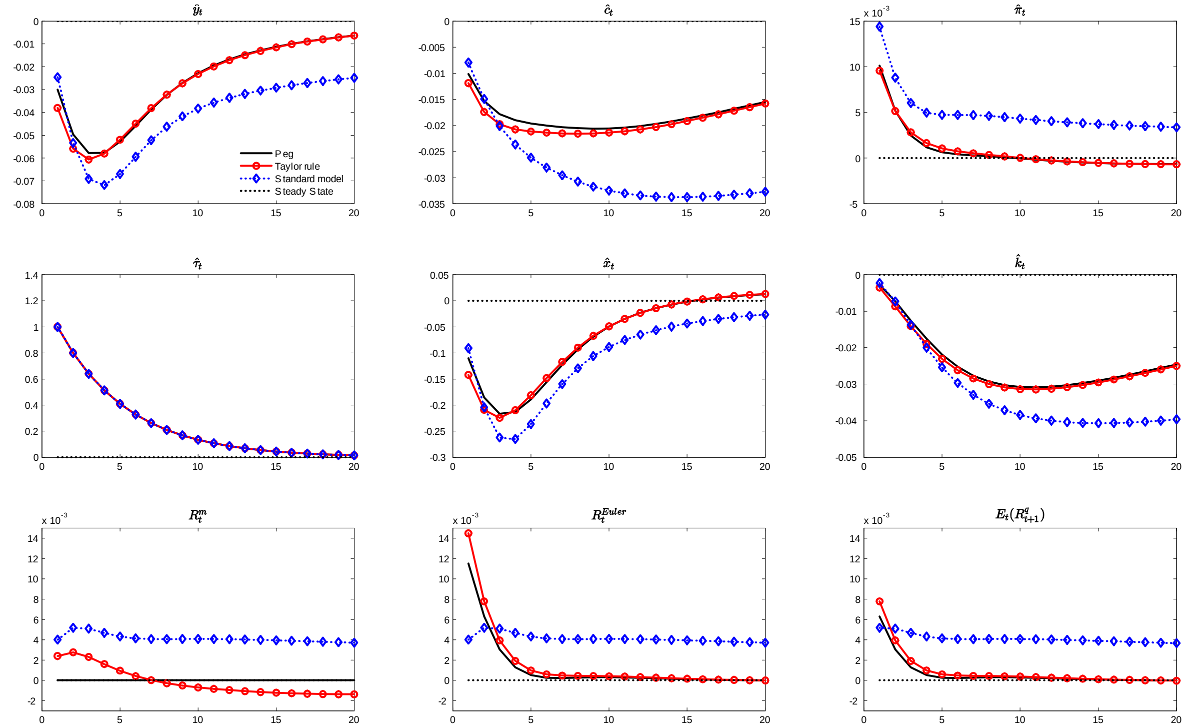

Figure 2: Responses to a positive income tax rate shock (in %)

equals the marginal rate of substitution.[14] For the applied parameter values, the equilibrium is locally determinate (see also Proposition 1). In all three cases, government spending has the usual wealth effect: higher public consumption crowds out private consumption while it leads to an increase in labor supply such that the fiscal multiplier is positive (as indicated by an increase in production). Real marginal costs then rise and therefore also inflation. Notably, even though the policy rate is pegged at zero, consumption is not crowded-in, which contrasts to the findings of Eggertsson (2011) and Christiano et al. (2011). Though the policy rate is at the ZLB (see solid lines), the marginal rate of intertemporal substitution increases, which can be seen from the responses of the Euler rate and the inflation rate.[15] Overall, the responses to higher government spending are not qualitatively affected by monetary policy pegging the policy rate rather adjusting it endogenously, while quantitatively we find that fiscal policy tends to be more expansionary under a peg but differences are moderate. On impact, fiscal multipliers are about 0.7 to 0.9. Fiscal multipliers are slightly higher if the policy rate is pegged at the ZLB but far from the values obtained by Eggertsson (2011), Christiano et al. (2011), and Woodford (2011).

Figure 2 shows responses to an autocorrelated ($\rho_\tau = 0.8$) increase in the labor income

[14] In this case, the policy rate is assumed to be set such that the money market constraint is not binding, $\eta_t = 0$.

[15] Notice that the mean policy rates differ between the model variants and is positive under a Taylor rule such that negative deviations from steady state do not imply negative negative policy rates.

tax rate. In accordance with the conventional view on tax effects, both, working time and consumption decline. As wages and real marginal costs increase, inflation rises, which causes the central bank to increase the policy rate if it applies a Taylor rule. All versions of the model unambiguously show that output declines regardless of monetary policy. Thus, the fact that the policy rate is pegged does again not affect the sign of the responses to higher taxes. This result is clearly at odds with Eggertsson's (2011) analysis, where an increase in the tax rate can be expansionary at the ZLB.

4 Conclusion

In this paper, we reconsider fiscal policy effects when the monetary policy rate is at the zero lower bound. We show that when the (real) policy rate is not identical to the marginal rate of substitution and nominal returns on other assets are strictly positive, fiscal policy shocks exert conventional effects also at the zero lower bound, i.e. government spending and income tax shocks increase inflation and reduce consumption. Hence, the fiscal multiplier is smaller than one, as in standard models when the ZLB is not binding.

5 References

Atkeson, A., and P.J. Kehoe, 2009, On the Need for a New Approach to Analyzing Monetary Policy, *NBER Macroeconomics Annual 2008 23*, 389-425.

Braun, R.A., L.M. Körber, and Y. Waki, 2012, Some Unpleasant Properties of Log-Linearized Solutions When the Nominal Rate Is Zero, Federal Reserve Bank of Atlanta, Working Paper 2012-5a.

Canzoneri M.B., R. E. Cumby, and B.T. Diba, 2007, Euler Equations and Money Market Interest Rates: A Challenge for Monetary Policy Models, *Journal of Monetary Economics* 54 , 1863-1881.

Christiano, J.L., M. Eichenbaum, and C.L. Evans, 2005, Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy* 113, 1-45.

Christiano, L., M. Eichenbaum, and S. Rebelo, 2011, When is the Government Spending Multiplier Large? *Journal of Political Economy* 119, 78-121.

Eggertsson, G.B., 2011, What Fiscal Policy Is Effective at Zero Interest Rates? *NBER Macroeconomic Annual 2010* 25, 59-112.

Erceg C.J. and J. Linde, 2014, Is There a Fiscal Free Lunch in a Liquidity Trap?, *Journal of the European Economic Association* 12, 73-107.

Groth, C. and Khan, H., 2010, Investment Adjustment Costs: An Empirical Assessment, *Journal of Money, Credit, and Banking* 42, 1469-1494.

Hoermann, M. and A. Schabert, 2014, A Monetary Analysis of Balance Sheet Policies, *The Economic Journal*, forthcoming.

Krishnamurthy, A., Vissing-Jorgensen, A., 2012, The Aggregate Demand for Treasury Debt, *Journal of Political Economy* 120, 233-267.

Mertens K. and M.O. Ravn, 2014, Fiscal Policy in an Expectations Driven Liquidity Trap, *Review of Economic Studies*, forthcoming.

Ohanian, L., 2011, Discussion of "What Fiscal Policy Is Effective at Zero Interest Rates?" by G. Eggertsson, *NBER Macroeconomic Annual 2010* 25, 125-137.

Simon, D., 1990, Expectations and the Treasury Bill-Federal Funds Rate Spread over Recent Monetary Policy Regimes, *Journal of Finance* 45, 567-577.

Woodford, M., 2011, Simple Analytics of the Government Expenditure Multiplier, *American Economic Journal: Macroeconomics* 3, 1-35.

A Appendix: Equilibrium

Definition 1 *A rational expectations equilibrium (REE) is a set of sequences* $\{c_t,\ y_t,\ n_t,\ x_t,\ k_t,\ w_t,\ \varrho_t,\ v_t,\ q_t,\ \lambda_t,\ m_t^R,\ m_t^H,\ b_t,\ b_t^T,\ mc_t,\ Z_{1,t},\ Z_{2,t},\ Z_t,\ s_t,\ \pi_t,\ R_t,\ R_t^{Euler},\ R_t^q\}_{t=0}^{\infty}$ *satisfying*

$$c_t = m_t^H + m_t^R,\ \text{if}\ \ R_t^{Euler} > 1,\quad \text{or}\ c_t \leq m_t^H + m_t^R,\ \text{if}\ \ R_t^{Euler} = 1, \tag{19}$$

$$b_{t-1}/\left(R_t^m \pi_t\right) = m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R,\ \text{if}\ \ R_t^{Euler} > R_t^m, \tag{20}$$

$$\text{or}\ b_{t-1}/\left(R_t^m \pi_t\right) \geq m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R,\ \text{if}\ \ R_t^{Euler} = R_t^m,$$

$$E_t \xi_{t+1} u_{c,t+1} \pi_{t+1}^{-1} = R_t E_t \left(R_{t+1}^m\right)^{-1} \xi_t u_{c,t} \pi_{t+1}^{-1}, \tag{21}$$

$$m_t^R = \Omega m_t^H, \tag{22}$$

$$\theta n_t^{\sigma_n} = u_{c,t}(1-\tau_t) w_t / R_t^{Euler}, \tag{23}$$

$$1/R_t^{Euler} = \beta E_t \left[\xi_{t+1} u_{c,t+1} / \left(\xi_t u_{c,t} \pi_{t+1}\right)\right], \tag{24}$$

$$w_t = mc_t \alpha n_t^{\alpha-1} k_{t-1}^{1-\alpha}, \tag{25}$$

$$R_t^q = P_t \left(v_t + \varrho_t\right) / \left(P_{t-1} v_{t-1}\right), \tag{26}$$

$$1 = \beta E_t \left[\left(\lambda_{t+1}/\lambda_t\right) \cdot \left(R_{t+1}^q / \pi_{t+1}\right)\right]. \tag{27}$$

$$\varrho_t = y_t - w_t n_t - x_t, \tag{28}$$

$$\lambda_t = \beta E_t \left[\xi_{t+1} u_{c,t+1} / \pi_{t+1}\right], \tag{29}$$

$$1 = q_t \left[\Lambda_t + \left(x_t/x_{t-1}\right) \Lambda_t'\right] - E_t \beta \left[\left(\lambda_{t+1}/\lambda_t\right) q_{t+1} \left(x_{t+1}/x_t\right)^2 \Lambda_{t+1}'\right], \tag{30}$$

$$q_t = \beta E_t \left[\left(\lambda_{t+1}/\lambda_t\right)\left((1-\alpha) mc_{t+1} \left(y_{t+1}/k_t\right) + (1-\delta)\, q_{t+1}\right)\right], \tag{31}$$

$$Z_{1,t} = \lambda_t y_t mc_t + \phi \beta E_t \pi_{t+1}^{\varepsilon} Z_{1,t+1}, \tag{32}$$

$$Z_{2,t} = \lambda_t y_t + \phi \beta E_t \pi_{t+1}^{\varepsilon-1} Z_{2,t+1}, \tag{33}$$

$$Z_t = \left[\varepsilon / \left(\varepsilon - 1\right)\right] Z_{1,t} / Z_{2,t}, \tag{34}$$

$$1 = (1-\phi) Z_t^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}, \tag{35}$$

$$s_t = (1-\phi) Z_t^{-\varepsilon} + \phi s_{t-1} \pi_t^{\varepsilon}, \tag{36}$$

$$y_t = n_t^{\alpha} k_{t-1}^{1-\alpha} / s_t, \tag{37}$$

$$y_t = c_t + x_t + g_t, \tag{38}$$

$$k_t = (1-\delta)\, k_{t-1} + x_t \Lambda_t, \tag{39}$$

$$b_t = b_t^T - m_t^H, \tag{40}$$

$$b_t^T = \Gamma b_{t-1}^T / \pi_t, \tag{41}$$

(where $u_{c,t} = (c_t - h c_{t-1})^{-\sigma}$, $\Lambda_t = 1 - \varsigma \frac{1}{2}\left(x_t/x_{t-1} - 1\right)^2$, $\Lambda_t' = -\varsigma\left(x_t/x_{t-1} - 1\right)$*), the transversality conditions, a monetary policy setting* $\{R_t^m \geq 1\}_{t=0}^{\infty}$, $\Omega_t > 0$, $\pi \geq \beta$, *and a fiscal policy setting* $\Gamma \geq 1$, *for given sequences* $\{\xi_t, g_t, \tau_t\}_{t=0}^{\infty}$ *and* $\{\widetilde{y}_t\}_{t=0}^{\infty}$ *(see below) and initial values* $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, $k_{-1} > 0$, $x_{-1} > 0$, *and* $s_{-1} \geq 1$.

The efficient output level $\widetilde{y}_t$ is jointly determined with the sequences $\{\widetilde{y}_t, \widetilde{n}_t, \widetilde{c}_t, \widetilde{k}_t, \widetilde{x}_t, \widetilde{q}_t\}_{t=0}^{\infty}$ satisfying $\theta \widetilde{n}_t^{1+\sigma_n} = \widetilde{u}_{ct} \alpha \widetilde{y}_t$, $\widetilde{y}_t = \widetilde{n}_t^{\alpha} \widetilde{k}_{t-1}^{1-\alpha}$, $\widetilde{y}_t = \widetilde{c}_t + \widetilde{x}_t$, $\widetilde{k}_t = (1-\delta)\, \widetilde{k}_{t-1} + \widetilde{x}_t \Lambda\left(\widetilde{x}_t/\widetilde{x}_{t-1}\right)$, $1 = \widetilde{q}_t \left[\Lambda\left(\widetilde{x}_t/\widetilde{x}_{t-1}\right) + \left(\widetilde{x}_t/\widetilde{x}_{t-1}\right) \Lambda'\left(\widetilde{x}_t/\widetilde{x}_{t-1}\right)\right] - E_t \beta \left[\xi_{t+1} \widetilde{u}_{c,t+1} (\xi_t \widetilde{u}_{c,t})^{-1} \widetilde{q}_{t+1} \left(\widetilde{x}_{t+1}/\widetilde{x}_t\right)^2 \Lambda'\left(\widetilde{x}_{t+1}/\widetilde{x}_t\right)\right]$, and $\widetilde{q}_t = \beta E_t[\xi_{t+1} \widetilde{u}_{c,t+1} (\xi_t \widetilde{u}_{c,t})^{-1}((1-\alpha)(\widetilde{y}_{t+1}/\widetilde{k}_t) + (1-\delta)\, \widetilde{q}_{t+1})]$, where $\widetilde{u}_{c,t} = (\widetilde{c}_t - h \cdot \widetilde{c}_{t-1})^{-\sigma}$ given $\widetilde{x}_{-1} > 0$ and $\widetilde{k}_{-1} > 0$.

If the money market constraint (1) is not binding, which would be the case when the policy rate equals the Euler equation rate, $R_t^m = R_t^{Euler}$ (see 17), the model as given in Definition 1 can be reduced to a conventional sticky price model with a cash-credit good distortion, where Ricardian equivalence holds and money holdings can separately be determined by (19) and (22) if $R_t^{Euler} > 1$. This version is used for the impulse responses of the "standard model" displayed by solid lines with diamonds in the Figures 1 and 2.

B Appendix: Simplified version

In this Appendix, we simplify the model by restricting the parameter values to $h = \rho_y = \rho_R = 0$, and $\alpha = \pi = \Gamma = 1$ and apply a log-linear approximation in the neighborhood of the steady state. We further assume that the mean policy rate and the inflation target are set at $R^m < \pi/\beta$, such that the multiplier on the money supply constraint (1) is strictly positive in the steady state (see 17). Hence, the money supply constraint is binding the neighborhood of the steady state.

Definition 2 *For $h = \rho_y = \rho_R = 0$, and $\alpha = \pi = \Gamma = 1$, a REE under a binding money supply constraint is a set of sequences $\{c_t,\ y_t,\ n_t,\ w_t,\ m_t^R,\ m_t^H,\ b_t,\ b_t^T,\ mc_t,\ Z_{1,t},\ Z_{2,t},\ Z_t,\ s_t,\ \pi_t,\ R_t,\ R_t^{Euler}\}_{t=0}^{\infty}$ satisfying (21)-(24), (32)-(36), (40), (41),*

$$c_t = m_t^H + m_t^R,\ b_{t-1}/\left(R_t^m \pi_t\right) = m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R, \tag{42}$$

$$w_t = mc_t,\ y_t = n_t/s_t,\ y_t = c_t + g_t, \tag{43}$$

and the transversality conditions, a monetary policy setting $\{R_t^m \geq 1\}_{t=0}^{\infty}$ according to (16), $\Omega_t > 0$, and $\pi \geq \beta$, and a fiscal policy setting $\Gamma \geq 1$, for given sequences $\{\xi_t, g_t, \tau_t\}_{t=0}^{\infty}$ and $\{\widetilde{y}_t\}_{t=0}^{\infty}$ and initial values $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, and $s_{-1} \geq 1$.

Log-linearizing the set of equilibrium conditions given in Definition 2, leads to

$$\widehat{mc}_t = \frac{\varrho_2}{\varrho_1} m^H \widehat{m}_t^H + \sigma E_t \widehat{m}_{t+1}^H + E_t \widehat{\pi}_{t+1} + \widehat{\xi}_t \left(1 - \rho_\xi\right) + \frac{\varrho_2}{\varrho_1}(1+\Omega)^{-1} g \widehat{g}_t + \frac{\tau^n}{1-\tau^n}\widehat{\tau}^n,$$

where $\varrho_1 = m^H + (1+\Omega)^{-1} g > 0$ and $\varrho_2 = \frac{\sigma_n + 1 - \alpha}{\alpha} > 0$ and using $E_t \widehat{\xi}_{t+1} = \rho_\xi \widehat{\xi}_t$, and to

$$\widehat{\pi}_t = (\beta + \chi) E_t \widehat{\pi}_{t+1} + \chi \frac{\varrho_2}{\varrho_1} m^H \widehat{m}_t^H + \chi \sigma E_t \widehat{m}_{t+1}^H + \chi \left(1 - \rho_\xi\right) \widehat{\xi}_t + \chi \frac{\varrho_2}{\varrho_1}(1-\Omega) g \widehat{g}_t + \frac{\chi \tau^n}{1-\tau^n}\widehat{\tau}^n.$$

Further using, $\widehat{b}_{t-1} - \widehat{R^m}_t - \frac{1+\Omega}{\Omega}\widehat{\pi}_t = \frac{1+\Omega}{\Omega}\widehat{m}_t^H - \frac{1}{\Omega}\widehat{m}_{t-1}^H$ and $\widehat{R^m}_t = \rho \widehat{\pi}_t$, the log-linearized version of the equilibrium conditions can be reduced to a system in $\widehat{\pi}_t$, $\widehat{m}_t^H$, and $\widehat{b}_t$ satisfying

$$\delta_1 E_t \widehat{\pi}_{t+1} + \delta_3 \widehat{b}_t + \delta_2 \widehat{m}_t^H = \widehat{\pi}_t - \left(\delta_\xi \widehat{\xi}_t + \delta_g \widehat{g}_t + \delta_\tau \widehat{\tau}_t^n\right), \tag{44}$$

$$\widehat{b}_t + \frac{1}{\Omega}\widehat{m}_t^H = -\frac{1+\Omega}{\Omega}\widehat{\pi}_t + \widehat{b}_{t-1} + \frac{1}{\Omega}\widehat{m}_{t-1}^H, \tag{45}$$

$$\widehat{b}_t - \widehat{m}_t^H = \rho \widehat{\pi}_t, \tag{46}$$

where $\delta_1 = \left(\beta + \chi(1-\sigma) - \chi\sigma\frac{\Omega}{1+\Omega}\rho\right) \gtreqless 0$, $\delta_2 = \chi\left(\frac{\varrho_2}{\varrho_1}m^H + \sigma\frac{1}{1+\Omega}\right) >$, $\delta_3 = \chi\sigma\frac{\Omega}{1+\Omega} >$, $\delta_\xi = \chi\left(1-\rho_\xi\right) > 0$, $\delta_g = \chi\frac{\varrho_2}{\varrho_1}(1+\Omega)^{-1}g > 0$ and $\delta_\tau = \chi\frac{\tau^n}{1-\tau^n} > 0$. Hence, all $\delta's$ except of δ_1 are strictly positive.

Proof of proposition 1. To establish the claims made in the proposition, we rewrite the model (44)-(46) in matrix form:

$$\begin{pmatrix} \delta_1 & \delta_3 & \delta_2 \\ 0 & 1 & \frac{1}{\Omega} \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} E_t\widehat{\pi}_{t+1} \\ \widehat{b}_t \\ \widehat{m}_t^H \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1+\Omega}{\Omega} & 1 & \frac{1}{\Omega} \\ \rho & 0 & 0 \end{pmatrix} \begin{pmatrix} \widehat{\pi}_t \\ \widehat{b}_{t-1} \\ \widehat{m}_{t-1}^H \end{pmatrix} + \begin{pmatrix} -\delta_\xi & -\delta_g & -\delta_\tau \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \widehat{\xi}_t \\ \widehat{g}_t \\ \widehat{\tau}_t^n \end{pmatrix}$$

where the characteristic polynomial of

$$A = \begin{pmatrix} \delta_1 & \delta_3 & \delta_2 \\ 0 & 1 & \frac{1}{\Omega} \\ 0 & 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1+\Omega}{\Omega} & 1 & \frac{1}{\Omega} \\ \rho & 0 & 0 \end{pmatrix}$$

is given by

$$H(X) = X^3 - \frac{\Omega\left(1+\delta_1+\delta_2+\delta_3+\rho\delta_2\right)+\delta_1+\delta_2+\delta_3-\rho\delta_3+1}{\delta_1\left(\Omega+1\right)}X^2 + \frac{\Omega-\rho\delta_3+\Omega\rho\delta_2+1}{\delta_1\left(\Omega+1\right)}X$$

Given that there are two backward-looking variables and one forward-looking variable, stability and uniqueness recquire exactly two stable roots. One root of $H(X)$ is $X_3 = 0$, and the others are given by the roots of

$$F(X) = X^2 - \frac{\Omega+\delta_1+\delta_2+\delta_3+\Omega\delta_1+\Omega\delta_2+\Omega\delta_3-\rho\delta_3+\Omega\rho\delta_2+1}{\delta_1\left(\Omega+1\right)}X + \frac{\Omega-\rho\delta_3+\Omega\rho\delta_2+1}{\delta_1\left(\Omega+1\right)}$$

where $H(X) = XF(X)$. For stability and uniqueness $F(X)$ has to be characterized by one stable and one unstable root. At $X = 0$, the sign of $F(X)$ equals the sign of δ_1

$$F(0) = \frac{\Omega-\rho\delta_3+\Omega\rho\delta_2+1}{\delta_1\left(\Omega+1\right)} = \frac{1}{\delta_1}\frac{1+\Omega+\Omega\rho\chi\frac{\varrho_2}{\varrho_1}m^H}{(1+\Omega)}$$

while $F(X)$ exhibits the opposite sign at $X = 1 : F(1) = -\frac{1}{\delta_1}\left(\delta_2+\delta_3\right)$. Note that δ_1 can be positive or negative. Consider first the case where $\delta_1 = \beta + \chi(1-\sigma) - \chi\sigma\frac{\Omega}{1+\Omega}\rho > 0 \Leftrightarrow$

$$\rho < \left(\frac{\beta}{\sigma\chi} + \frac{1-\sigma}{\sigma}\right)\frac{1+\Omega}{\Omega} \tag{47}$$

Given that $\sigma \geq 1$ and $\beta < 1$, we know that δ_1 is then strictly smaller than one. Hence, $F(1) < 0$ and $F(0) > 1$, which implies that exactly one remaining root must be unstable and the stable root is strictly positive. Now consider the second case where $\delta_1 = \beta + \chi(1-\sigma) - \chi\sigma\frac{\Omega}{1+\Omega}\rho < 0 \Leftrightarrow \rho > \frac{\beta+\chi(1-\sigma)}{\chi\sigma\Omega/(1+\Omega)}$, such that $F(1) > 0$ and $F(0) < 0$. We then know that there is at least one stable root between zero and one. To establish a condition which ensures that there is exactly one root, we further use

$$F(-1) = \frac{2(\delta_1+1)(\Omega+1) + (1+\Omega+2\Omega\rho)\delta_2 + (1+\Omega-2\rho)\delta_3}{_1(\Omega+1)}$$

Rewriting the numerator with $\delta_1 = \beta + \chi(1-\sigma) - \chi\sigma\frac{\Omega}{1+\Omega}\rho$, $\delta_2 = \chi\frac{\varrho_2}{\varrho_1}m^H + \chi\sigma\frac{1}{1+\Omega}$ and $\delta_3 = \chi\sigma\frac{\Omega}{1+\Omega}$ we get the condition

$$2\left(\beta + \chi(1-\sigma) - \chi\sigma\frac{\Omega}{1+\Omega}\rho + 1\right)(\Omega+1) + (1+\Omega+2\Omega\rho)\chi\frac{\varrho_2}{\varrho_1}m^H + 2\chi(1+\Omega) > 0$$

which ensures that $F(0)$ and $F(-1)$ exhibit the same sign implying that there is no stable root between zero and minus one, if but not only if

$$\rho \leq \left(\frac{1+\beta}{\chi\sigma} + \frac{1-\sigma}{\sigma}\right)\frac{1+\Omega}{\Omega} \tag{48}$$

where the RHS of (48) is strictly larger than the RHS of (47). Using that $\frac{1+\Omega}{\Omega} < 1$, the following condition is sufficient for local equilibrium stability and uniqueness

$$\rho \leq \frac{1+\beta}{\chi\sigma} + \frac{1-\sigma}{\sigma} \tag{49}$$

which establishes the claim made in the proposition. ■

Proof of proposition 2. For $\Omega \to \infty$, the set of equilibrium conditions (44)-(46) can, with (22), be rewritten as

$$\delta_1 E_t\widehat{\pi}_{t+1} + \delta_3\widehat{b}_t + \delta_2\widehat{m}_t^R = \widehat{\pi}_t - \left(\delta_\xi\widehat{\xi}_t + \delta_g\widehat{g}_t + \delta_\tau\widehat{\tau}_t^n\right), \tag{50}$$

$$\widehat{b}_{t-1} - (1+\rho)\widehat{\pi}_t = \widehat{m}_t^R, \tag{51}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{52}$$

where $\delta_1 = (\beta + \chi(1-\sigma) - \chi\sigma\rho) \gtreqless 0$, $\delta_2 = \chi\frac{\varrho_2}{\varrho_1}m^R > 0$, $\delta_3 = \chi\sigma > 0$, $\delta_\xi = \chi\left(1-\rho_\xi\right) > 0$, $\delta_g = \chi\frac{\varrho_2}{\varrho_1}g > 0$ and $\delta_\tau = \chi\frac{\tau^n}{1-\tau^n} > 0$. We aim at identifying the impact responses to fiscal policy shocks. For this we assume that (18) is satisfied, which ensures existence and uniqueness of a locally stable solution. We can therefore apply the following solution form for the system (50)-(52)

$$\widehat{\pi}_t = \gamma_{\pi b}\widehat{b}_{t-1} + \gamma_{\pi g}\widehat{g}_t + \gamma_{\pi\tau}\widehat{\tau}_t + \gamma_{\pi\xi}\widehat{\xi}_t,$$
$$\widehat{b}_t = \gamma_b\widehat{b}_{t-1} + \gamma_{bg}\widehat{g}_t + \gamma_{b\tau}\widehat{\tau}_t + \gamma_{b\xi}\widehat{\xi}_t,$$
$$\widehat{m}_t^R = \gamma_{mb}\widehat{b}_{t-1} + \gamma_{mg}\widehat{g}_t + \gamma_{m\tau}\widehat{\tau}_t + \gamma_{m\xi}\widehat{\xi}_t$$

and $\widehat{R}_t^m = \gamma_R \widehat{b}_{t-1} + \gamma_{Rg}\widehat{g}_t + \gamma_{R\tau}\widehat{\tau}_t + \gamma_{R\xi}\widehat{\xi}_t$. In what follows, we identify the undetermined coefficients for $\widehat{\xi}_t = 0$. Using these generic solutions, conditions (50)-(52) imply

$$\begin{aligned}
0 &= (\delta_1\gamma_{\pi b}\gamma_b + \delta_3\gamma_b + \delta_2\gamma_{mb} - \gamma_{\pi b})\,\widehat{b}_{t-1} \\
&\quad + \left(\delta_1\gamma_{\pi b}\gamma_{bg} + \delta_1\gamma_{\pi g}\rho_g + \delta_3\gamma_{bg} + \delta_2\gamma_{mg} - \gamma_{\pi g} + \delta_g\right)\widehat{g}_t \\
&\quad + \left(\delta_1\gamma_{\pi b}\gamma_{b\tau} + \delta_1\gamma_{\pi\tau}\rho_g + \delta_3\gamma_{b\tau} + \delta_2\gamma_{m\tau} - \gamma_{\pi\tau} + \delta_\tau\right)\widehat{\tau}_t^n, \\
0 &= ((1+\rho)\,\gamma_{\pi b} + \gamma_{mb} - 1)\,\widehat{b}_{t-1} + \left((1+\rho)\,\gamma_{\pi g} + \gamma_{mg}\right)\widehat{g}_t + ((1+\rho)\,\gamma_{\pi\tau} + \gamma_{m\tau})\,\widehat{\tau}_t, \\
0 &= (\gamma_b + \gamma_{\pi b} - 1)\,\widehat{b}_{t-1} + \left(\gamma_{bg} + \gamma_{\pi g}\right)\widehat{g}_t + (\gamma_{b\tau} + \gamma_{\pi\tau})\,\widehat{\tau}_t.
\end{aligned}$$

Hence, we get the conditions $\delta_1\gamma_{\pi b}\gamma_{bg} + \delta_3\gamma_{bg} + \delta_1\gamma_{\pi g}\rho_g - \gamma_{\pi g} + \delta_2\gamma_{mg} + \delta_g$

$$\gamma_{\pi b} = \delta_1\gamma_{\pi b}\gamma_b + \delta_3\gamma_b + \delta_2\gamma_{mb},\ 1 = (1+\rho)\,\gamma_{\pi b} + \gamma_{mb},\ 1 = \gamma_b + \gamma_{\pi b}, \tag{53}$$

$$-\delta_2\gamma_{mg} = (\delta_1\gamma_{\pi b} + \delta_3)\,\gamma_{bg} + \left(\delta_1\rho_g - 1\right)\gamma_{\pi g} + \delta_g,\ \ -\gamma_{mg} = (1+\rho)\,\gamma_{\pi g},\ \gamma_{bg} = -\gamma_{\pi g}, \tag{54}$$

$$-\delta_2\gamma_{m\tau} = (\delta_1\gamma_{\pi b} + \delta_3)\,\gamma_{b\tau} + \left(\delta_1\rho_g - 1\right)\gamma_{\pi\tau} + \delta_\tau,\ \ -\gamma_{m\tau} = (1+\rho)\,\gamma_{\pi\tau},\ \gamma_{b\tau} = -\gamma_{\pi\tau}. \tag{55}$$

Start with the three conditions in (53) and eliminate $\gamma_{\pi b}$ with $\gamma_{\pi b} = 1 - \gamma_b$, to give

$$0 = (\delta_1\gamma_b - 1)(1-\gamma_b) + \delta_3\gamma_b + \delta_2\gamma_{mb},\ 1 = (1+\rho)(1-\gamma_b) + \gamma_{mb},$$

and use $1 - (1+\rho)(1-\gamma_b) = \gamma_{mb}$, to get

$$0 = (\delta_1\gamma_b - 1)(1-\gamma_b) + \delta_3\gamma_b + \delta_2\,(1 - (1+\rho)(1-\gamma_b))$$

which leads to the quadratic equation

$$(-\delta_1)\,\gamma_b^2 + (\delta_1 + \delta_3 + \delta_2\,(\rho+1) + 1)\,\gamma_b - (\rho\delta_2 + 1) = 0.$$

Defining $K(X) = X^2 - \frac{\delta_1+\delta_3+\delta_2(\rho+1)+1}{\delta_1}X + \frac{\rho\delta_2+1}{\delta_1}$, such that $K(0) = \frac{\rho\delta_2+1}{\delta_1}$, $K(1) = -\frac{\delta_2+\delta_3}{\delta_1}$ and $K(-1) = \frac{2\delta_1+\delta_2+\delta_3+2\rho\delta_2+2}{\delta_1}$, and using $\delta_1 = (\beta + \chi\,(1-\sigma) - \chi\sigma\rho)$, gives

$$K(-1) = \frac{2\,(\beta + \chi\,(1-\sigma) - \chi\sigma\rho + 1) + (1+2\rho)\,\delta_2 + \delta_3}{\delta_1}$$

which shows that $K(-1)$ has the same sign as $K(0)$ if (49) holds. Then, there exists exactly one stable root $\gamma_b \in (0,1)$. Hence, we can use

$$\gamma_{mb} = 1 - (1+\rho)(1-\gamma_b)\,,\ \ \gamma_{\pi b} = 1 - \gamma_b > 0,$$

to assess the fiscal policy effects with the three conditions in (54), which reveal that the impact responses of inflation and real money exhibit different signs. Eliminating γ_{bg} with

$\gamma_{bg} = (1+\rho)^{-1}\gamma_{mg}$, gives

$$\begin{aligned}\gamma_{mg} &= -\frac{\delta_g}{(\delta_1\gamma_{\pi b}+\delta_3)(1+\rho)^{-1} - (1+\rho)^{-1}\left(\delta_1\rho_g - 1\right) + \delta_2} \\ &= -\frac{(1+\rho)\,\delta_g}{\delta_1\left(\gamma_{\pi b}-\rho_g\right)+\delta_3+1+(1+\rho)\,\delta_2}\end{aligned}$$

and using $\delta_1 = (\beta + \chi(1-\sigma) - \chi\sigma\rho)$, $\delta_2 = \chi\frac{\varrho_2}{\varrho_1}m^R$, and $\delta_3 = \chi\sigma$, leads to

$$\gamma_{mg} = -\frac{(1+\rho)\,\delta_g}{(\beta+\chi(1-\sigma)-\chi\sigma\rho)\left(\gamma_{\pi b}-\rho_g\right)+\chi\sigma+1+(1+\rho)\,\delta_2}.$$

Note that $\left|\gamma_{\pi b}-\rho_g\right| < 1$, and that we assumed $\beta+\chi(1-\sigma)-\chi\sigma\rho+1>0$ (see 18) which ensures a strictly positive denominator given that $\sigma \geq 1$. Now consider the three conditions in (55) to identify the tax effects. Using $\gamma_{b\tau} = -\gamma_{\pi\tau}$ and $\gamma_{\pi\tau} = -(1+\rho)^{-1}\gamma_{m\tau}$ and $0 = (\delta_1\gamma_{\pi b}+\delta_3)\gamma_{b\tau} + \left(\delta_1\rho_g-1\right)\gamma_{\pi\tau} + \delta_2\gamma_{m\tau} + \delta_\tau$, one gets

$$0 = -(1+\rho)^{-1}\gamma_{m\tau}\left(\delta_1\rho_g - 1 - \delta_1\gamma_{\pi b} - \delta_3\right) + \delta_2\gamma_{m\tau} + \delta_\tau,$$

which leads to

$$\gamma_{m\tau} = -\frac{(1+\rho)\,\delta_\tau}{\delta_1\left(\gamma_{\pi b}-\rho_g\right)+\delta_3+1+(1+\rho)\,\delta_2}$$

where the denominator is identical to the one of γ_{mg}. Hence, real balances and therefore consumption falls on impact in response to higher government spending and higher taxes, while inflation increases. ■

C Appendix: A negative preference shock

Consider the set of equilibrium conditions (50)-(52), as well as $\widehat{m}_t^R = \widehat{b}_t - \widehat{R^m}_t$, $\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t$, and $\widehat{R^m}_t = \rho\widehat{\pi}_t$. Proceeding as in the proof of proposition 2, we can then derive the solution coefficients for $\widehat{\xi}_t \neq 0$:

$$\begin{aligned}\gamma_{R\xi} &= -\rho(1+\rho)^{-1}\gamma_{m\xi} > 0,\ \gamma_{\pi\xi} = -(1+\rho)^{-1}\gamma_{m\xi} > 0, \\ \gamma_{m\xi} &= -\frac{(1+\rho)\,\delta_\xi}{\delta_1\left(\gamma_{\pi b}-\rho_g\right)+\delta_3+1+(1+\rho)\,\delta_2} < 0,\ \gamma_{b\xi} = -\gamma_{\pi\xi} < 0.\end{aligned}$$

Hence, a sufficiently large decrease in $\widehat{\xi}_t$, i.e. $\widehat{\xi}_t < -(R^m-1)/\gamma_{R\xi}$, causes the nominal interest rate to reach its zero lower bound, $R_t^m = 1$.