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# Forward Guidance in a Simple Model with a Zero Lower Bound

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## Abstract

In this paper we present a three period setup to model central bank forward guidance in a liquidity trap. We analyze the role of long-run and short-run price stickiness under discretion and commitment in a straightforward and intuitive way. Despite the impact of price rigidity on welfare being non-linear, losses under discretion are lowest with perfectly flexible prices. We show why the zero lower bound may still be binding even long after the shock has gone and characterize conditions when a commitment to hold nominal rates at zero for an extended period is optimal. We then introduce government spending and show that under persistently low policy rates optimal government spending becomes more front-loaded, while procyclical austerity fares worse than discretionary government spending.

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# 1  Introduction

Forward guidance is seen as a handy tool to steer the real rate of interest and real activity when the nominal policy rate is stuck at the zero lower bound (ZLB). Using the framework of Benigno (2009), our aim is to analyze –in the spirit of Barro and Gordon (1983)– the ingredients for credible and feasible forward guidance within a traceable setup. Thereby, we reconsider recent advances on a theoretical foundation of forward guidance, which can, for example, be found in Eggertsson and Woodford (2003) and Werning (2012). We characterize the role of long and short run price stickiness under discretion and commitment. In our model the impact of price rigidity on welfare is non–linear, but losses under discretion are always lowest with perfectly flexible prices. We show that credibility depends on the feasibility of *"irresponsible"* (Krugman, 1998) monetary easing and analyze why the zero lower bound may still be binding long after any shock has abated. We characterize conditions when a commitment to hold nominal rates at zero for an extended period is optimal.

Recently, Cochrane (2013) argues that –due to nominal indeterminacy– the New Keynesian framework exhibits multiple equilibria with different price paths, some of them with mild inflation and no output loss during a liquidity trap. We characterize the optimal price path in our model and show that price stickiness eliminates price level indeterminacy under optimal policy.

We then introduce government spending as additional policy tool. We show that countercyclical spending is always optimal. When policy rates are zero for an extended period of time, government spending should become more front–loaded. However, since fiscal spending affects nominal rates, the credibility of an announced government spending path might be constrained by the ZLB even after the adverse shock fully abated. Finally, we show that procyclical fiscal policy always results in welfare losses that are even higher than under discretionary policy.

Our model shows that the effectiveness of forward guidance strongly depends on the ability of credible commitment to future activities –a commitment which most central banks have been reluctant to make. However, provided a central bank has sufficient credibility, forward guidance –combined with other unconventional policy measures– can contribute to mitigating the problem of the ZLB successfully. Figure 1 provides suggestive evidence for the US Fed. The solid line indicates the effective policy rate for the US as estimated by Wu and Xia (2014), adjusting the actual rate for unconventional policy measures. The dashed line shows estimates for the natural real rate by Barsky, Justiniano, and Melosi (forthcoming). The vertical line indicates March 2009, the first time the Fed announced *"exceptionally low levels of the federal funds rate for an extended period."* While other unconventional measures have also contributed to driving the effective policy rate closer to the natural real rate, the Fed's credible forward guiding announcements

since early 2009 have been a key factor (Campell, Evans, Fisher, Justiniano, Calomiris, and Woodford, 2012). Understanding the ingredients for successful forward guidance both with and without fiscal support is at the core of this paper.

**Figure 1:** Forward Guidance in the US
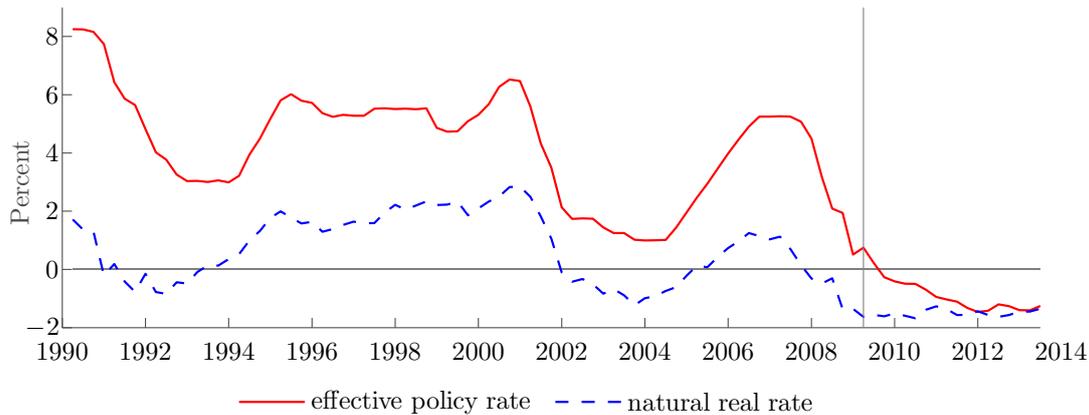


*Notes:* The time series for the effective policy rate (solid line) is taken from Wu and Xia (2014). The time series for the natural real rate (dashed line) is reproduced from Barsky, Justiniano, and Melosi (forthcoming). The vertical line indicates March 2009, the first time when the Fed announced "*exceptionally low levels of the federal funds rate for an extended period.*"

The reminder of the paper is organized as follows: Section 2 outlines our model. Section 3 analysis the problem of dynamic inconsistency and the role of price rigidities on welfare losses. Section 4 derives the optimal forward guidance paths. Section 5 introduces fiscal spending into our model and Section 6 concludes.

## 2   Model

We consider a three-period setup. Periods 1 and 2 can be thought of as the short run and period 3 as the long run, i.e. after period 3 variables do not change. The households' optimization problem is given by

$$\max_{\{C_t,N_t\}_{t=1}^3} \mathbb{E}_1\left[\sum_{t=1}^3 \Big(\frac{1}{1+\rho_t}\Big)^{t-1}\left(\frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{N_t^{1+\varphi}}{1+\varphi}\right)\right]$$

$$s.t.$$

$$P_1C_1 + B_1 = W_1N_1 + T_1$$

$$P_2C_2 + B_2 = W_2N_2 + (1+i_1^S)B_1 + T_2$$

$$P_3C_3 = W_3N_3 + (1+i_2^S)B_2 + T_3$$

where $\rho_t$ is the stochastic discount rate, $\sigma$ is the elasticity of inter-temporal substitution, $\varphi$ characterizes the elasticity of labor supply, $C_t$ is consumption, $N_t$ are hours worked and $P_t$ is the price level. Households save via the purchase of short-term (one period) bonds, $B_t$, which yield interest $i_t^S$. $W_t$ is the nominal wage rate and $T_t$ are lump sum net transfers including firms' profits. It is straightforward to derive the log-linear aggregate-demand curves

$$y_t - y^\star = \mathbb{E}_t[y_{t+1} - y^\star] - \sigma \left( i_t^S - [\mathbb{E}_{[}p_{t+1}] - p_t] - \rho_t \right), \quad t \in \{1, 2\} \tag{1}$$

where $y_t \equiv \log Y_t$, $p_t \equiv \log P_t$ and $y^\star$ denoting the efficient (log–)level of production.

Firms have mass one. We are interested in the interaction between short–run and long–run price stickiness. To keep the model simple, we assume that in period 0 a share $\alpha_1$ of firms has to fix its prices for at least two periods. These firms cannot adjust in periods 1 and 2. With probability $\lambda$ each of the $\alpha_1$-type firms may even not be able to adjust their prices in period 3. The parameter $\lambda$ allows us to vary the degree of long–run rigidity in period 3 independent of rigidities in the other periods. A share $\alpha_2$ of firms exhibits short–run price stickiness. These firms set their price always one period in advance (at $t-1$) before observing shocks occurring during period $t$. The remaining $1 - (\alpha_1 + \alpha_2)$ firms can adjust their prices freely at any time. We later consider a demand shocks occurring only in period 1. So the $\alpha_2$–types, if perfectly anticipating the future price path, will set the same price level in period 2 and 3 as the free–adjusters. Firms production technology is given by $Y_t(i) = A_t N_t(i)$, $\forall i \in [0, 1]$, and the good market is monopolistic competitive such that $Y_t(i) = (P_t(i)/P_t)^{-\theta} Y_t$ with $\theta$ being the elasticity of substitution between the continuum of goods. Given this pricing scheme aggregate (log-) supply is described by

$$y_t - y^\star = \frac{1}{\kappa_t}(p_t - p^\star), \quad t \in \{1, 2, 3\} \tag{2}$$

where $\kappa_1 = \frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}(\frac{1}{\sigma} + \varphi)$, $\kappa_2 = \frac{1-\alpha_1}{\alpha_1}(\frac{1}{\sigma} + \varphi)$ and $\kappa_3 = \frac{1-\alpha_1\lambda}{\alpha_1\lambda}(\frac{1}{\sigma} + \varphi)$. Since $\lim_{\alpha_1 \to 0} \kappa_2 = \lim_{\alpha_1 \to 0} \kappa_3 = +\infty$ but $\lim_{\alpha_1 \to 0} \kappa_1 \neq +\infty$ there will be no output gaps in period 2 and 3 if prices are perfectly flexible. In period 1, however, an output gap emerges independently of $\alpha_1$ since also the $\alpha_2$–type firms have their period 1 prices set to $p^\star$.

Monetary policy is characterized by announcing a future price path $\{p_2, p_3\}$ to forward guide expectations. The central bank's objective is to minimize the quadratic loss function derived from a second order Taylor approximation of the utility function

$$\mathcal{L}_1 = \frac{1}{2}\mathbb{E}_1 \left[ \sum_{t=1}^{3} \left( \prod_{j=0}^{t-1} \frac{1}{1 + \rho_j} \right) \left\{ (y_t - y^\star)^2 + \frac{\theta}{\kappa_t}(p_t - p^\star)^2 \right\} \right] \tag{3}$$

where $p^\star$ is the steady state price level $\alpha_1$–type firms have anchored their prices to and $y^\star$ is the flexible price output. When minimizing welfare losses the central bank is constrained by the New Keynesian IS–curves, Equation (1) and Phillips–curves, Equations (2).

For the simulation exercises in Section 4 and 5 we choose a standard calibration with $\beta = 0.99$, $\sigma = \varphi = 1$, $\theta = 5$. We choose $\alpha_2$ to be small to allow for high $\alpha_1$ when $\alpha_1 \to 1 - \alpha_2$: $\alpha_2 = 0.1$. Somewhat arbitrarily we choose $\alpha_1 = 0.25$, such that in period 1 approximately one third of firms cannot adjust their prices, and $\lambda = 1$ as the impact of these parameters on optimal policy is analyzed in the next section. When introducing government spending, we assume that in efficient equilibrium $G^\star / Y^\star = 0.2$ and the inverse intertemporal elasticity of substitution of government spending $\eta_g = 1$.

# 3 The problem of dynamic inconsistency

To provide the simplest framework for our liquidity trap analysis, we consider the following thought experiment: before period 1 the economy is in its steady state and the central bank has been expected to stabilize prices perfectly: $\mathbb{E}_0[p_t] = p^\star$, $t \in \{1, 2, 3\}$. Following Eggertsson (2006) we assume that in period 1 a negative time preference shock, $\rho_1$, with $\rho_1 < 0 < \bar{\rho} = \rho_2$, hits the economy and drives it to the zero lower bound. There is no persistence in the shock, so the economy will revert back to steady state in period 2. But solely by cutting the interest rate down to zero, the central bank cannot prevent a recession in period 1 since this would require a negative nominal rate. It can, however, announce to raise the price levels in the following periods above $p^\star$ in order to lower the current real rate of interest and thus to stimulate current consumption even when the nominal policy rate remains stuck at zero. To perfectly stabilize the economy in the first period the central bank would need to credibly announce a price level of $\bar{p}_2 = p^\star + |\rho_1|$ for period 2. Such a policy, however, will never be optimal commitment strategy: raising $p_2$ above $p^\star$ causes inefficiencies and thus welfare loss next period. The optimal commitment strategy is to promise to raise $p_2$ only so much that the marginal loss in period 2 (from accepting a price $p_2 > p^\star$) will be just equal to the marginal gains in period 1 (from preventing $p_1$ to fall too far below $p^\star$). Credibility is a crucial feature of forward guidance: if agents have marginal doubt in the central banks willingness to implement the announced path the strategy unravels. This is shown in Figure 2 where, for the sake of simplicity, we assume that $p_3 = p^\star$ and that the nominal rate of interest, $i_2^S$ and $i_1^L$ can be set consistently.

In Figure 2, point $C$ with $(p_2^\star, y_2^C)$ characterizes the optimal commitment strategy. However, this commitment solution suffers from the well understood problem of dynamic inconsistency: the promise to implement the commitment path is not credible. As soon as the shock has gone and time preference reverts back to $\bar{\rho}$ in period 2, the central bank has an incentive to renege on its promises. After all, at that stage, aggregate demand reverts to normal, so there is no longer any reason to stimulate the economy. The promise to raise the price level anyway implies the central bank is nevertheless willing to shift prices and output beyond target levels. Suppose the central bank wants to raise next period's price level to $p_2^\star$ by shifting the AD curve in Figure 2 upwards to $AD^C$. Ex–ante, from

period 1's perspective, only the prices of $\alpha_1$–types have already been fixed. So the ex–ante relevant AS–curve is the line $DC$ described by Equation (2). Thus, long–run price rigidity as captured by $\alpha_1$ allows forward guidance to raise both price level and output in period 2, despite being fully anticipated. If agents trust that promise, the expected price level next period will be $\mathbb{E}_1[p_2] = p_2 = \alpha_1 p^\star + (1 - \alpha_1)p_2^\star$, bringing the economy to point $C$. Once period 2 has been reached, however, the central bank faces a new, flatter $AS^{ex-post}$–curve, since ex post now also prices of the $\alpha_2$–types are fixed and thus price rigidities are stronger. The new AS–curve intersects the flexible price output at $\hat{p}_2 = \frac{\alpha_1}{\alpha_1+\alpha_2}p^\star + \frac{\alpha_2}{\alpha_1+\alpha_2}p_2^\star$. Ex–post, the central bank has an incentive to ignore past statements and to decrease prices instead. Given sticky prices it will choose point $A$ rather than $C$.

**Figure 2:** Dynamic inconsistency in period 2
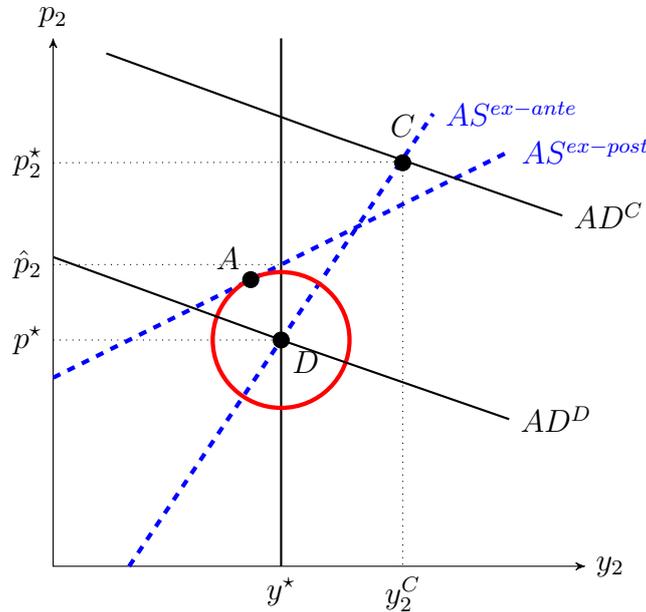


As long as firms have marginal doubt about the central bank's commitment to stick to its promise, they will anticipate this incentive already in period 1 and will charge a price below $p_2^\star$, thereby reducing $p_2$. For the same reason private consumers will not trust that the central bank is willing to implement a high rate of inflation in period 2. Being afraid that instead the real rate of interest will stay high, they prefer to save rather than to spend in period 1. So the strategy unravels. In the end the unique discretionary equilibrium is to implement $\{p^\star; y^\star\}$ at point D. Therefore, under discretion the expectation channel breaks down and the central bank is not able to credibly promise any excess inflation in period 2 ($p_2^D = p_3^D = p^\star$ and $y_2^D = y_3^D = y^\star$). There is no way to attenuate the adverse effects of the severe recession in period 1 and monetary policy, constrained by the ZLB, will remain too tight. Without any commitment to future activities the shock will hit the economy full tilt with the real rate being too high, $\rho_1 = r_1^n < r_1^D = \frac{\sigma\kappa_1}{1+\sigma\kappa_1}\rho_1 < 0$,

and strong deflation $p_1^D - p^\star = \frac{\kappa_1 \sigma}{1 + \kappa_1 \sigma} \rho_1 < 0$.[1] Only under credible forward guidance as postulated by Assumption 1 in the next section the central bank can guide agents' expectations.

In our setup, deflation in period 1 gets worse without any intervention the lower the degree of period 1 price stickiness, $\alpha \equiv \alpha_1 + \alpha_2$. This result is also found by Werning (2012). However, since with flexible prices output volatility is low and deviations of the price level from target have a small weight in the welfare function, aggregate welfare losses are lowest with perfect price flexibility as shown in Figure 3. This result is in strong contrast to Werning's *Proposition 2*. In our discrete time setup we identify, moreover, a non–linearity in the effect of price rigidity on welfare loss. In contrast to Werning, in our model discretionary welfare losses $\mathcal{L}_1^D = \frac{1}{2} \frac{1 + \theta \kappa_1}{(1 + \sigma \kappa_1)^2} (\sigma \rho_1)^2$ are decreasing in $\alpha$ only for high degrees of price stickiness. This threshold is reached for $\alpha \geq \bar{\alpha} = \frac{1 + \sigma \varphi}{2(1 - \frac{\sigma}{\theta}) + \sigma \phi}$. Since $\alpha \in [0, 1]$, $\alpha$ will always be below $\bar{\alpha}$ for $\bar{\alpha} \geq 1$ or equivalently for $2\sigma \geq \theta$. Generally, aggregate welfare losses will start to decrease in the degree of price stickiness only for a sufficiently high degree of inter–firm competition. Nevertheless, even in this case welfare losses always exceed the losses under fully flexible prices. The reason for this non–linearity is the following:

1. an increase in price rigidity, $\alpha$, makes the AS–curve flatter increasing output volatility for given price deviations $p_1^D - p^\star$. This induces higher welfare losses.

2. In contrast, an increase in $\alpha$ reduces price volatility, which improves welfare.

3. However, an increase in $\alpha$ also raises the weight, $\theta/\kappa_1$, of price deviations in the welfare loss function (see Equation (3)).[2]

For $\alpha \in [0, \sigma/(1 + \sigma)]$ the third effect dominates the second effect since the increase of the welfare weight is initially higher than the reduction in price volatility. Therefore, for $\alpha$ low enough, the first and the aggregate effect of 2. and 3. work into the same direction and welfare losses rise in $\alpha$. However, the second effect is more convex than the third effect and thus the more $\alpha$ increases the stronger becomes the former relative to the latter (at $\alpha = \sigma/(1 + \sigma)$ both effects are equal). For $\alpha > \sigma/(1 + \sigma)$ the aggregate welfare effect of 2. and 3. turns positive, attenuating the negative effect of higher output volatility. Since for further increases in $\alpha$ the aggregate positive effect (2. + 3.) on welfare is more convex than the negative effect (1.) the former effect gradually catches up and at $\alpha = \bar{\alpha}$ the total effect of price rigidity on welfare starts turning positive.

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[1] The real rate of interest is defined as $r_t = i_t - (\mathbb{E}_t p_{t+1} - p_t)$; $r_t^n$ denotes the natural real rate.

[2] Note that the weight on output deviations is normalized to unity.

**Figure 3:** Price Rigidities and Welfare Loss
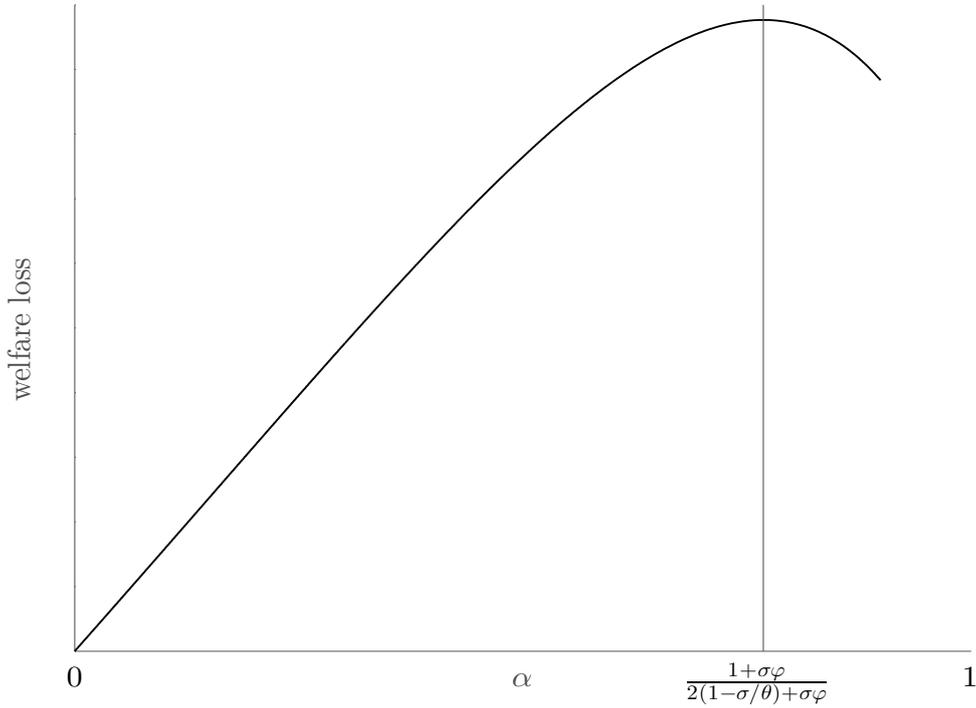


# 4 Forward Guidance in a liquidity trap

To derive the optimal price path under forward guidance, we assume from now on that forward guidance is credible.

**Assumption 1.** *The central bank's announced price path $\{p_2, p_3\}$ is credible in the sense that*

$$\mathbb{E}_t[p_{t+1}] = p_{t+1}, \ t \in [1, 2]$$

The central bank is assumed to be able to guide the aggregate price level perfectly via the announcements.[3] To solve for optimal policy in a liquidity trap we minimize Equation

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[3]Dropping the expectation operator, the expected price level in periods 2 and 3 is given by $p_2 = \alpha_1 p^\star + (1 - \alpha_1)p_2^\star$ and $p_3 = \alpha_1 \lambda p^\star + (1 - \alpha_1 \lambda)p_3^\star$, respectively.

(3) s.t. Equations (1) and (2), Assumption 1 and $i_1^S = 0$. The solution is given by

$$
0 = \frac{1 + \theta\kappa_1}{\kappa_1^2}(p_1 - p^\star) + \frac{1}{1 + \rho_1}\frac{(1 + \theta\kappa_2)(1 + \kappa_1\sigma)}{\kappa_1\kappa_2(1 + \kappa_2\sigma)}(p_2 - p^\star) + \ldots
$$

$$
\ldots + \frac{1}{1 + \rho_1}\frac{1}{1 + \bar{\rho}}\frac{(1 + \theta\kappa_3)(1 + \kappa_1\sigma)}{\kappa_1\kappa_3(1 + \kappa_3\sigma)}(p_3 - p^\star),
$$

$$
p_1 - p^\star = \frac{\kappa_1(1 + \kappa_2\sigma)}{\kappa_2(1 + \kappa_1\sigma)}(p_2 - p^\star) + \frac{\kappa_1\sigma}{1 + \kappa_1\sigma}\rho_1,
$$

$$
i_2^S = \bar{\rho} + \frac{1 + \kappa_3\sigma}{\kappa_3\sigma}(p_3 - p^\star) - \frac{1 + \kappa_2\sigma}{\kappa_2\sigma}(p_2 - p^\star)
$$

(4)

The first equation of (4) requires optimal policy to equalize marginal welfare losses across time. Thereby, monetary policy is constrained by the remaining equations. Since the ZLB is binding in period 1, i.e. $i_1^S = 0$, there will be positive co–movement between $p_1$ and $p_2$, as a higher $p_2$ increases inflation between these periods and thus lowers the real rate which stimulates demand in period 1. The short–term nominal rate between periods 2 and 3, $i_2^S$, is not necessarily zero as shown in the third equation of (4).

In period 1, a discount factor shock disturbs the economy, driving the natural rate below zero. With the ZLB restricting the short–term policy rate, a recession is triggered. While under discretion the economy reverts back to steady state in period 2, optimal policy dampens period 1 recession via promising excess inflation in period 2 forcing the real rate of interest in period 2 below its natural level $r_2^n = \bar{\rho}$. Optimal policy allocates period welfare losses optimally across time. Optimally, in the long run (in period 3) the price level reverts to $p^\star$. This, however, requires deflation between period 2 and 3. To offset deflation, the central bank needs to lower the real rate in period 2 below the natural rate by cutting the nominal rate sufficiently. With households having rational expectations, the real rate of interest is determined by the Fisher equation. Thus, under credible price level guidance the nominal rates have to adjust consistently to the announced price path to satisfy the Fisher equation and to implement certain required real rate. This imposes a crucial constraint on credible forward guidance: the central bank cannot promise to implement arbitrarily high deflation between periods 2 and 3 as this might require negative nominal interest rates.

Let us first assume that the shock in period 1, $\rho_1$, is weak enough such that the ZLB will not be binding in period 2.

**Assumption 2a.** *The discount factor shock $\rho_1$ is small enough such that under optimal policy the ZLB is not binding on $i_2^S$, i.e.*[4]

$$
|\rho_1| \leq \left(1 + \frac{1}{1 + \rho_1}\frac{1 + \theta\kappa_2}{1 + \theta\kappa_1}\left(\frac{1 + \kappa_1\sigma}{1 + \kappa_2\sigma}\right)^2\right)\bar{\rho}
$$

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[4]Whether the ZLB is still binding in period 2 depends on $\rho_1$ and on the other model parameters, in particular the degree of price stickiness caputred by $\kappa_1$ and $\kappa_2$. Their effects are discussed further below.

Under Assumption 2a we can solve (4) for optimal policy analytically. As long as the ZLB is not binding in period 2 the optimal price target in period 3 is $p_3 = p^\star$ for the following reason: as long as optimal policy is able to dampen the recession via excess inflation in period 2 only, there is no need to deviate in period 3 from the target $p^\star$. In that case, any change in the price level in period 3 would lead only to a corresponding change in the nominal rate $i_2^S$ according to the third equation in (4), having no real effect. Thus, price deviations from $p^\star$ in period 3 would only induce additional welfare losses due to price distortions. Therefore, unconstrained optimal forward guidance implements $p_3 = p^\star$. Figure 4 shows the optimal policy paths compared to the discretionary solution given the baseline parameter calibration and $\rho_1 = -0.01$.

**Figure 4:** Optimal vs Discretion Policy
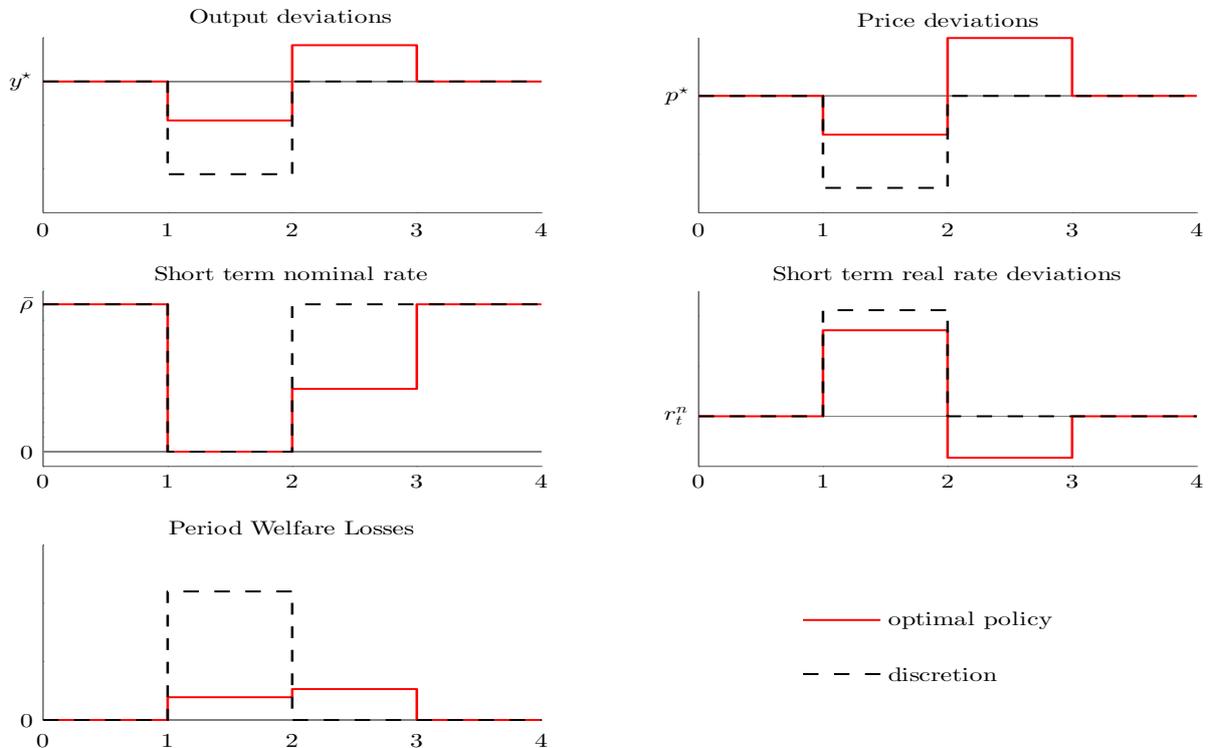


*Notes:* Unconstrained commitment solution for baseline calibration and $\rho_1 = -0.01$ to ensure that the ZLB is not binding for $i_2^S$.

So, under Assumption 2a second best policy will bring the economy back to steady state in period 3. With $p_3 = p^\star$ but $p_2 > p^\star$ optimal policy triggers deflationary expectations between periods 2 and 3, which needs to be offset by lowering $i_2^S$ to ensure that $r_2 < r_2^n = \bar{\rho}$. The central bank, however, cannot credibly promise arbitrary high price levels in $t = 2$: for large enough shocks, this would require a negative $i_2^S$. Thus, the ZLB becomes binding also in period 2 for large negative $\rho_1$ and for low degrees of price stickiness, $\alpha_1$, as shown in Figure 5 for $\rho_1 = -0.02$.

**Figure 5:** Effect of $\alpha_1$ on optimal policy
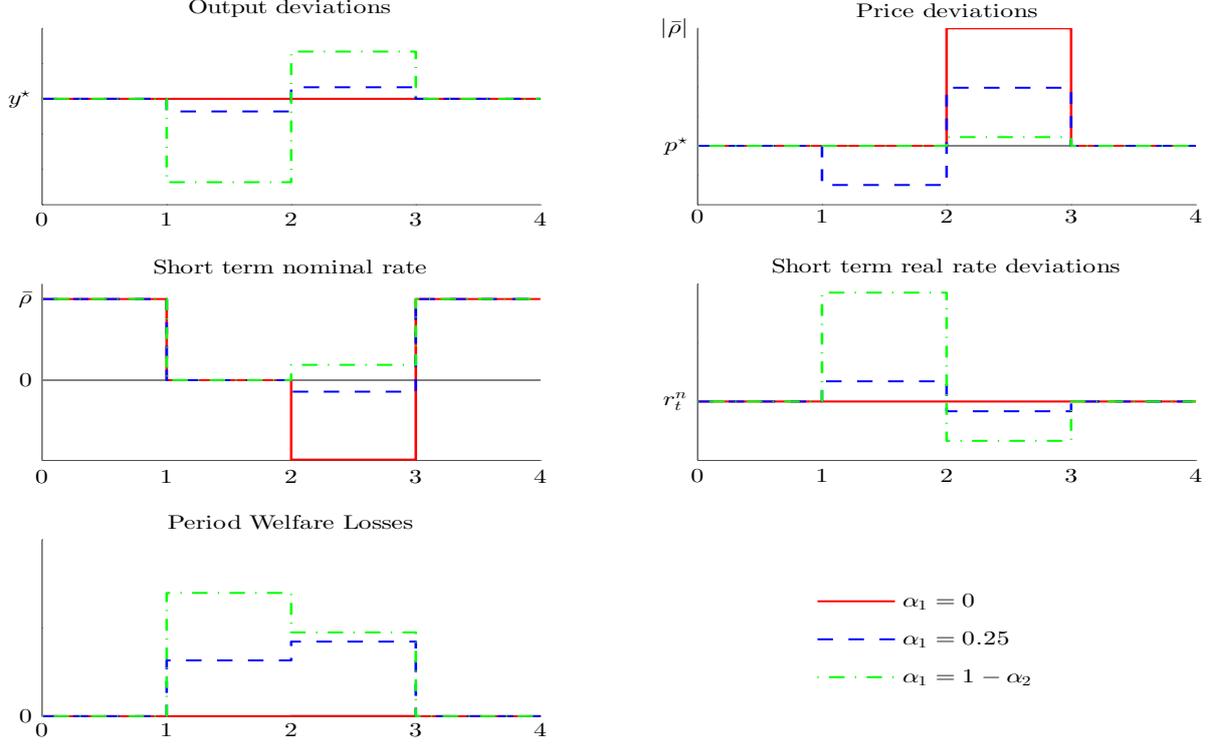


*Notes:* All parameters except $\alpha_1$ are kept at their baseline calibration and $\rho_1 = -0.02$ to ensure that for $\alpha_1 = 0$ the ZLB is violated for $i_2^S$.

The lower the degree of price stickiness, the stronger the excess inflation the central bank aims to implement in period 2. The transmission mechanism is straightforward: the lower the degree of price stickiness, i.e. the smaller the fraction of firms that fixed their prices at $p^\star$, the lower the weight of price deviations on welfare losses for $t = 2, 3$.[5] Therefore, announcing future price deviations gets less costly for monetary policy. But note that to implement high excess inflation the short–run nominal rate may turn negative in period 2.[6] If so, the announced optimal price path $\{p_2, p_3\}$ cannot be credible since agents anticipate that the nominal rate of interest cannot adjust consistently to this announcement.

Moreover, we see that aggregate welfare losses decrease monotonically with the degree of price stickiness. The reason is that in our model welfare losses due to price deviations occur only because some firms have fixed their prices already in period 0 and cannot adjust thereafter. If the fraction of these firms goes to zero, price deviations in period 2 and 3 are costless and thus monetary policy can stabilize period 1 perfectly. In the absence of long–run price stickiness (i.e. for $\alpha_1 = 0$), the zero lower bound is no longer

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[5]Note that $\lim_{\alpha_1 \to 0} \frac{\theta}{\kappa_2} = \lim_{\alpha_1 \to 0} \frac{\theta}{\kappa_3} = 0$.

[6]In this calibration we neglect Assumption 2a for expository purposes and solve for optimal policy without imposing the constraint $i_2^S \geq 0$.

a binding constraint even in the first period. In that case, the first best outcome can be implemented by raising $p_2$ to $p^\star + |\rho|$. Due to nominal indeterminacy, the price level $p_3$ is indetermined for $\alpha_1 = 0$.

As shown in the third panel of Figures 5 and 6 for a high degree of price–flexibility and/or large enough shocks, the optimal commitment strategy is not feasible since nominal rates cannot be negative in period 2. In that case Assumption 2a is violated. So we now consider the case that the ZLB is a binding constraint also for period 2.

**Assumption 2b.** *The discount factor shock $\rho_1$ is large enough and/or the degree of price stickiness is low such that under optimal policy the ZLB will be binding also in period 2, violating Assumption 2a. In that case $i_2^S = 0$*

The severity of the shock drives nominal rates to zero and thus restricts monetary authorities in implementing the optimal commitment price path. With the feasible amount of deflation between periods 2 and 3 being limited, policy is now restricted to be third best, requiring $p_3 > p^\star$ in order to be able to credibly promise excess inflation in period 2. Thus, in the third best solution the monetary authority must accept welfare losses also in period 3 to stabilize the discount factor shock.

Using $i_2^S = 0$ in (4) allows us to solve for constrained optimal policy analytically. Figure 6 shows optimal policy with the ZLB being binding in period 2 compared to unconstrained optimal policy and the discretionary solution for $\rho_1 = -0.05$. Under constrained optimal policy, forward guidance can provide less stimulation in period 1. The maximum downward jump in the price path from $t = 2$ to $t = 3$ is constrained by the ZLB on $i_2^S$ as the central bank cannot provide enough nominal ease to make any larger drop credible to agents. This can be seen when looking at the unconstrained solution –that is an optimal policy bringing the economy back to initial steady state in period 3 while announcing strong excess inflation in period 2. The drop in the price level required is so large that it drives $i_2^S$ far into negative territory. As agents anticipate that this is not feasible, the announced price path is thus not credible and the monetary authority can only implement the constrained best solution which induces higher aggregate welfare losses. Thus, third best policy has to keep the short–run nominal rate at the ZLB even after the shock has gone. Note that this is no direct consequence of the shock itself but of the intertemporal trade-off between raising $p_2$ to attenuate the recession and the corresponding deflation between period 2 and 3.

**Figure 6:** Optimal Policy and the ZLB in period 2
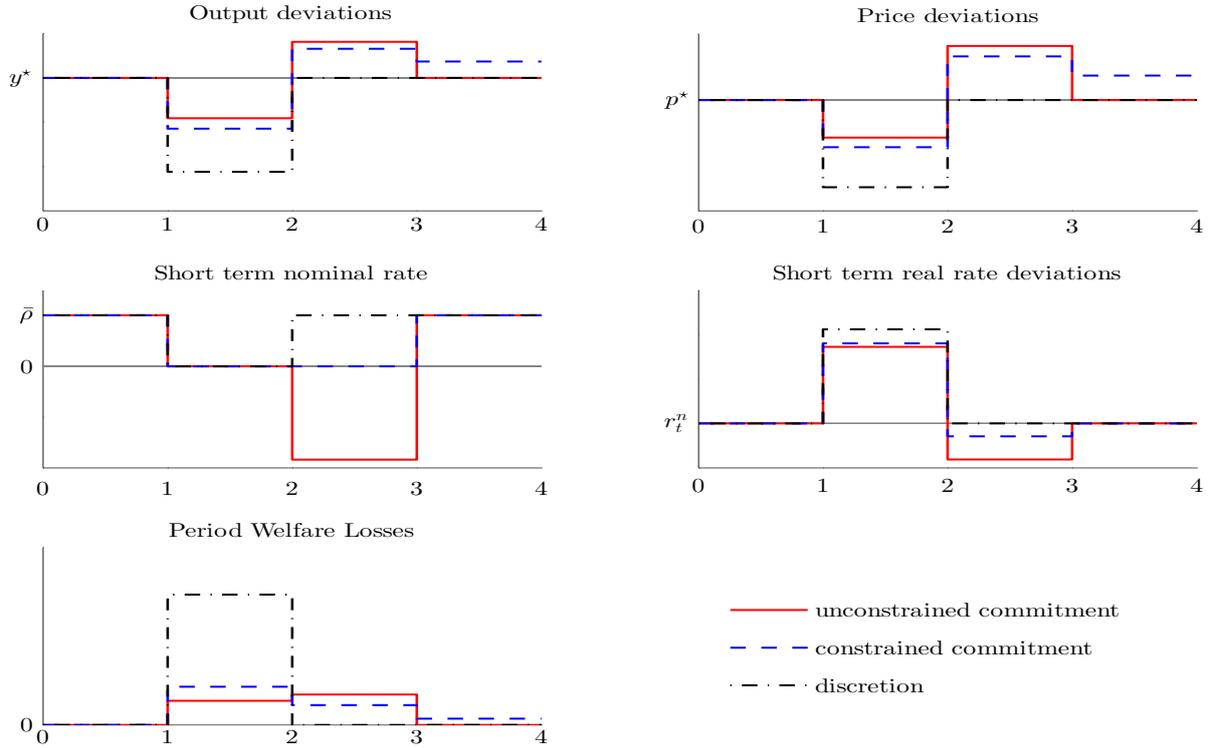


*Notes:* Constrained optimal solution for baseline calibration and $\rho_1 = -0.05$ to ensure that the ZLB is binding for $i_2^S$.

Whereas under unconstrained optimal policy the price path is decreasing between $t = 2$ and $t = 3$, this is not necessarily the case for constrained forward guidance. If period 1 and period 2 prices are very rigid ($\alpha_1 \rightarrow 1 - \alpha_2$) but period 3 prices are very flexible, constrained optimal policy can mostly affects period 1 price expectations via period 3 announcements. The optimal price path is then increasing between periods 2 and 3. However, the optimality condition to allocate welfare losses over time still determines the optimal price level in period 3 uniquely.

Extending our model to $n$ periods, for $n$ large enough the ZLB will at some point cease being a binding constraint as the deflation required can be spread across multiple periods. So, for large enough $n$ third best policy will bring the price level again back to $p^\star$.

The effect of price stickiness on constrained optimal policy is similar to before. Again, the lower the degree of price stickiness in the model, the more excess inflation will be triggered under constrained forward guidance. For $\alpha_1 = 0$ the economy can be stabilized perfectly, without any welfare losses occurring over time as in that case the welfare weight on price deviations from period 2 on is zero. The higher $\alpha_1$ the less accommodative policy is and for $\alpha_1 = 1 - \alpha_2$ barely any excess inflation will be announced. But due to a very

flat AS–curve even these small deviations will be very costly as they imply strong output deviations.

Recently, Cochrane (2013) argued that most results usually found in New Keynesian models during a liquidity trap are artifacts of an arbitrary equilibrium choice. To this end he introduces additional equilibria, identified by different steady state inflation rates that persist once the ZLB stops binding. These equilibria feature price paths that deviate arbitrarily from the old equilibrium path. Our model allows to elaborate on the question how the degree of price stickiness in period 3, the "long–run" period, influences (constrained) optimal policy. To this end, we adjust the parameter $\lambda$ that determines the degree of price stickiness in period 3 only. For $\lambda = 0$ the 3 price level in period 3 is perfectly flexible. We find that (constrained) optimal policy determines $p_3$ uniquely. The lower $\lambda$ the larger are price deviations in period 3, as the welfare weight of deviations approaches zero ($\lim_{\lambda \to 0} \frac{\theta}{\kappa_3} = 0$). Consequently, monetary policy can announce stronger excess inflation for period 2, given that $p_3$ can deviate more strongly. Even for $\lambda$ close to zero no arbitrarily large price deviations in period 3 do occur as excess stimulation cannot be optimal as well. The price level $p_3$ will be indetermined only for $\lambda = 0$. Hence, with only marginal price rigidities (constrained) optimal policy eliminates price level indeterminacy and thus does not support arbitrary equilibrium choice.

# 5 Optimal Government Spending

Up to this point policy could only stimulate during a zero interest rate environment by forward guiding expectations about the future price path. We now introduce government spending as an additional commitment device to attenuate the adverse effects of the liquidity trap. To this end, we follow Woodford (2011) and add additively separable government consumption to the household's utility function. To see how government spending works in our model it is illustrative to consider the modified (log-linear) aggregate demand curves

$$y_t - y^\star = \mathbb{E}_t[y_{t+1} - y^\star] + \mathbb{E}_t[\hat{g}_t - \hat{g}_{t+1}] - \tilde{\sigma}\left[\frac{i_t^S - \rho_t}{1 + \bar{\rho}} - \mathbb{E}_t[(p_{t+1} - p^\star) - (p_t - p^\star)]\right] \quad (5)$$

with $\hat{g}_t \equiv \frac{G_t - G^\star}{Y^\star}$ and $\tilde{\sigma} \equiv \sigma(y^\star - g^\star)$. In the absence of shocks, optimal government spending will be at some steady state level $g^\star$. To stimulate period $t$ production fiscal policy has two instruments at hand: first, it can raise $\hat{g}_t$ to induce a direct demand effect on output and to make up for any private demand shortfall. Second, it can announce a decreasing government spending path between period $t$ and $t + 1$ ($\hat{g}_t - \mathbb{E}_t\hat{g}_{t+1} > 0$). This increases marginal utility of consumption of households in period $t$ relative to period $t+1$, as agents anticipate that future private consumption will be high due to less crowding–out. Hence, in addition to the announcement of a price level path, the credible commitment to

some optimal path for government spending allows to attenuate the shock both directly and indirectly.

Given the time preference shock, at first sight it might seem optimal to cut government spending in the initial period in the same way as consumers cut current spending –after all, the social planner should internalize the time preference shock. With current real market rates being high, calling for austerity measures might be seen as the optimal response. But realizing that shadow rates are low, optimal policy will be characterized by intertemporal countercyclical spending shifts. It will be optimal to shift the path of fiscal policy relative to the optimal first best path by raising government spending (lowering taxes) in the first (the liquidity trap) period relative to the second period (the period required to stimulate consumption by keeping the real rate below the natural rate). It pays to aim at positive (negative) additional spending during the period when the real rate is above (below) the natural rate, as long as the social planner realizes that this helps to bring the market rate closer to the shadow (natural) rate. Since even under commitment, it is never optimal for monetary policy to bring the real rate down to the natural rate during the liquidity trap period, additional instruments can always improve upon pure monetary policy. In that sense, macro "trumps" public finance.

Let us derive analytically the optimal government spending path under Assumptions 1–2b and $i_1^S = 0$. Under full commitment over both, the future price and government spending path, the joint monetary and fiscal authority now minimizes

$$\mathcal{L}_1^G = \frac{1}{2}\mathbb{E}_1\left[\sum_{t=1}^{3}\left(\prod_{j=0}^{t-1}\frac{1}{1+\rho_j}\right)\left\{\varphi(y_t - y^\star)^2 + \eta_g\hat{g}_t^2 + \eta_u(y_t - y^\star - \hat{g}_t)^2 + \frac{\theta(1+\varphi)}{\sigma\kappa_t}(p_t - p^\star)^2\right\}\right]$$

(6)

s.t.

$$p_1 - p^\star = \frac{\kappa_1(\kappa_2 + \tilde{\sigma})}{\kappa_2(\kappa_1 + \tilde{\sigma})}\mathbb{E}_1[p_2 - p^\star] + \frac{\kappa_1}{\kappa_1 + \tilde{\sigma}}(\hat{g}_1 - \mathbb{E}_1[\hat{g}_2]) - \frac{i_1^S - \rho_1}{(1+\bar{\rho})(\kappa_1 + \tilde{\sigma})}$$

(7)

$$p_2 - p^\star = \frac{\kappa_2(\kappa_3 + \tilde{\sigma})}{\kappa_3(\kappa_2 + \tilde{\sigma})}\mathbb{E}_2[p_3 - p^\star] + \frac{\kappa_2}{\kappa_2 + \tilde{\sigma}}(\hat{g}_2 - \mathbb{E}_2[\hat{g}_3]) - \frac{\kappa_2\tilde{\sigma}}{(1+\bar{\rho})(\kappa_2 + \tilde{\sigma})}[i_2^S - \bar{\rho}]$$

(8)

with $\eta_u \equiv 1/\sigma$, $\eta_g \equiv -\frac{\partial^2 U(\cdot)}{\partial G_t^2}\left(\frac{\partial U(\cdot)}{\partial G_t}\right)^{-1}Y^\star$. Equation (6) is derived from a second order approximation of the extended utility function.[7] Equations (7) and (8) represent the AS–AD equilibrium in periods 1 and 2, respectively, derived from Equations (2) and (5). We assume that government spending is financed via the lump–sum transfers $T_t$.

It is straightforward to show that, independent of commitment and the ZLB, optimal government spending is countercyclical in the sense that $\hat{g}_t$ is inversely proportional to $(p_t - p^\star)$. Thus, government consumption, which, unlike private consumption, can be perfectly adjusted by policy independently of the current market rate, is a tool to smooth output fluctuations by leaning against the wind.

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[7]See Woodford (2003), chapter 6, for details on the derivation.

Unconstrained optimal policy features a countercyclical government spending path with all variables returning to their efficient levels in $t = 3$ (see solid line in Figure 7). The increase in government spending in period 1 makes up partially for the shortfall in private consumption and the credible commitment to relatively lower government spending in the future induces households to shift consumption again into period 1 via lower marginal utility in future periods. However, as above, implementing the unconstrained commitment path is feasible only as long as the nominal interest rate is non–negative in period 2. If, however, the adverse shock is large enough the ZLB will again be binding also in $t = 2$. The reason can be seen in equation (8): given the optimal price level path, mitigating the ZLB might require $\hat{g}_2 - \hat{g}_3$ to be positive, i.e. procyclical fiscal spending in period 2 or deviations from $g^\star$ in $t = 3$. This cannot be optimal and hence government spending will not eliminate the possibility of a binding ZLB in period 2 in the presence of large shocks. In this case, as shown in Figure 7 for large adverse shocks, monetary policy is again limited in its ability to credible promise excess inflation for $t = 2$ (third panel in Figure 7), such that, as in Section 4, the drop from $p_2$ to $p_3$ is limited under constrained optimal policy (dashed line in Figure 7).

**Figure 7:** Optimal vs Discretionary Policy
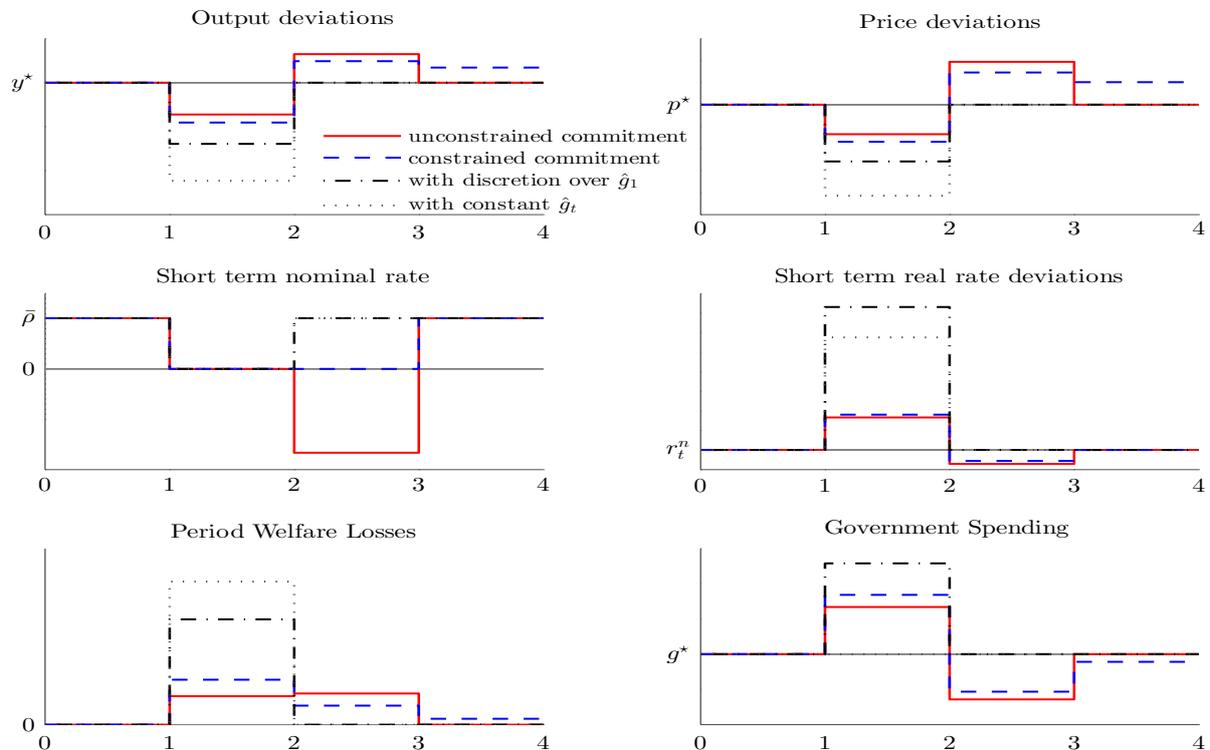


*Notes:* For this simulation $\rho_1 = -0.05$.

However, government spending can partially make up for the short–fall of monetary pol-

icy by providing additional stimulus in the first period compared to the unconstrained solution. Note, however, that under constrained forward guidance the indirect stimulative effect of government spending, via low marginal utility of private consumption in the second period, is also constrained by the ZLB in $t = 2$. Since, via Equation (8), $\partial i_2^S / \partial (\hat{g}_2 - \hat{g}_3) > 0$ an upward sloping government spending path between period 2 and 3 exhibits additional downward pressure on the nominal interest rate. Thus, the credible amount of future austerity that can be promised in $t = 1$ is limited and $\hat{g}_3$ has to deviate below $g^\star$ to allow for enough countercyclical spending in $t = 2$. In that sense, under constrained optimal policy the short–run direct effect of countercyclical government spending is even more important. If the central bank keeps the policy rate at the ZLB for an extended period of time even after the shock abated, this should optimally be accompanied with stronger front–loaded countercyclical fiscal policy. Any short–fall in fiscal stimulus, e.g. due to procyclical austerity measures, will impose welfare costs onto the economy as we show later.

Let us finally turn to discretionary policies. We consider two different scenarios: first, we assume that monetary policy cannot commit to future activities and government spending is fully inactive (dotted line in Figure 7). Second, we assume that both monetary and fiscal cannot commit but that fiscal policy reacts optimally to the slump in period 1, for which no commitment is needed (ragged line in Figure 7). Clearly, without any commitment possible and hands of monetary policy being tied by the ZLB, fiscal policy can help to increase aggregate demand to attenuate the recession. The demand effect of increasing government spending and the deceasing government spending path offsets the slump partially even without any credible promise to future excess inflation.

During the recent crisis there have been calls for austerity spending even when policy rates are close to or at zero. To see the effects of such a policy we now analyze the case that the fiscal government, just like the household, takes the real rate as given and adjusts consumption accordingly, i.e. $G_t = C_t \; \forall t \in \{1, 2, 3\}$. Thus, with a high real rate at the ZLB, government consumption will be shifted into the future inducing a procyclical spending path and austerity. We assume that households and monetary policy are aware of this behavior and that monetary policy satisfies Assumption 1. In this case, forward guidance is again limited to the announcement of the future price level path.

The dashed lines in Figure 8 show optimal forward guidance given passive government behavior. For illustration, we consider the case of a small shock so that the ZLB is not binding in the second period.[8] Government spending is now procyclical with high fiscal consumption when the real rate is low and vice versa. This policy turns out to be worse in terms of welfare than optimal unconstrained policy (solid line in Figure 8). The intuition is straightforward: procyclical government spending with austerity in the recession period amplifies economic fluctuations both through direct demand effects and via creating the

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[8]The results are similar for a binding ZLB in $t = 2$.

incentive for households to further postpone consumption until period 2 when marginal utility is high.

Remarkably, procyclical fiscal policy fares also worse than the discretionary solution with active government spending in period 1 (ragged line in Figure 8). Since monetary policy internalizes the effects of its price level decisions onto government behavior, it is more reluctant to trigger a boom in $t = 2$ as procyclical fiscal policy would amplify the output effects of excess inflation. Despite lower inflation in $t = 2$ the real rate in period 1 drops sharply as output and prices deteriorate under procyclical fiscal spending. This partially dampens the drop in consumption and government spending. The recession in $t = 1$ remains, however, severe. This, together with further fluctuations in periods 2 and 3 induces higher aggregate welfare losses onto the economy than under discretionary monetary and fiscal policy. In the latter case, losses in period 1 are high, but no additional losses occur in later periods . It is important to note that this result holds qualitatively independently of the calibration of $\eta_u$ and $\eta_g$; it is independent of the weight of output and government spending fluctuations in the welfare loss function.

**Figure 8:** Austerity policy
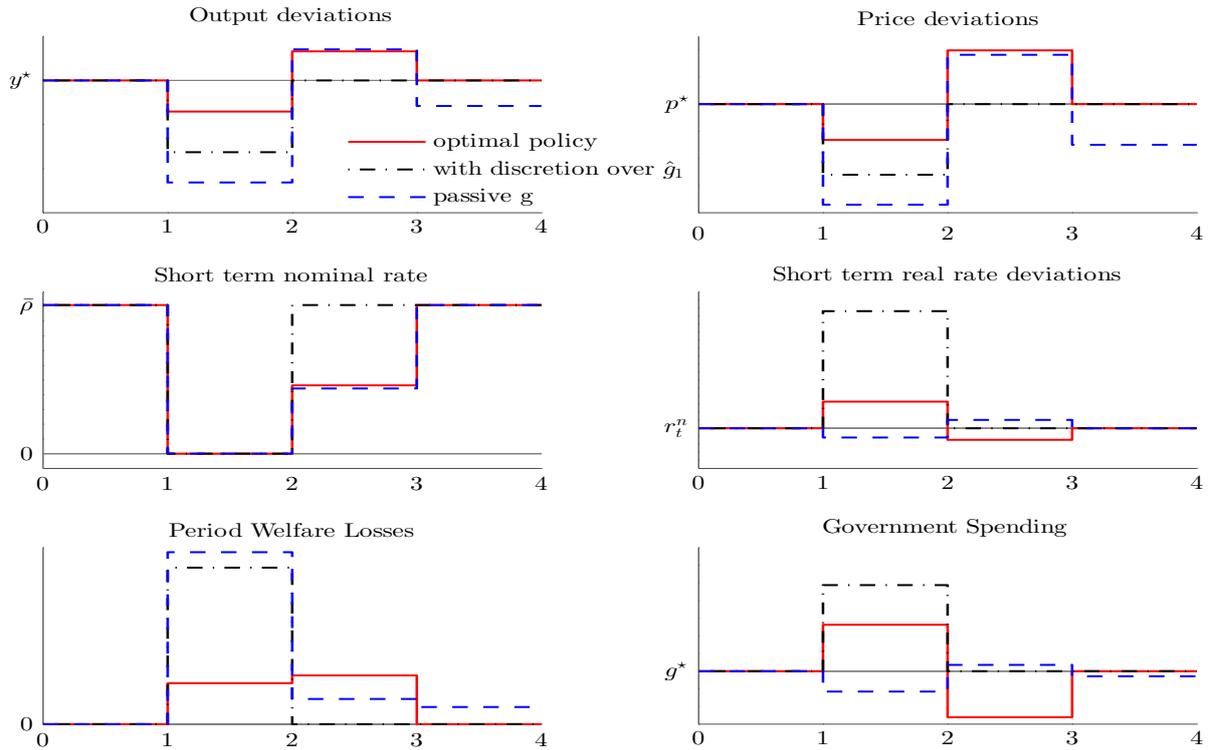


*Notes:* For this simulation $\rho_1 = -0.01$.

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# 6   Conclusion

The implications of our model are manifold: first and well understood, forward guidance is only effective if central banks can credibly commit to future actions (Assumption 1).

Second, if forward guidance is credible, price expectations are fully determined by the announced price path. This introduces an additional constraint to policy making via the Fisher equation. The nominal interest rate must be set consistently with the announced price path. Optimal policy needs to induce deflationary expectations between periods 2 and 3. Therefore, the monetary authority faces a trade off between promising excess inflation from period 1 to period 2 and the deflation required to bring the price level back to target in period 3. So the amount of excess inflation in period 2 may be constrained by the ZLB even after the shock has already faded away. Understanding this mechanism is of key importance for credible forward guidance.

Third, we have shown that price stickiness eliminates price level indeterminacy under optimal policy. Thus, the equilibrium choice, once the discount factor shock abated and the ZLB ceases binding, is not arbitrary but well defined in our model. Optimal forward guidance policy aims to bring the price level back to the target price level $p^\star$ in period 3. Therefore, and unlike argued by Cochrane (2013), this equilibrium choice is not arbitrary but optimal in our model.

Fourth, for high levels of stickiness welfare under discretion may increase with the degree of price stickiness. But in strong contrast to Werning (2012), in our model welfare losses under discretion are always lowest when prices are perfectly flexible.

Finally, we extended the model to allow for fiscal policy as commitment device. With the ZLB being binding, the market real rate of interest is above the natural (shadow) rate in period 1. So it is optimal to shift the path of fiscal policy relative to the optimal first best path by raising government spending (lowering taxes) in the first relative to the second period. In contrast, procyclical austerity policy induces even higher welfare losses than discretionary policy.

So far we have kept the nominal anchor $p^\star$ fixed over time. An interesting extension would be to allow $p^\star$ to grow over time, adjusting to a certain inflation target $\pi^\star$. This would allow us to elaborate on the argument brought forward by Blanchard, Dell'Ariccia, and Mauro (2010) and Ball (2013). They argue that a higher inflation target may increase welfare by reducing the probability of hitting the ZLB in the future. Within our setup, if the $\alpha_1$–type firms are able to adjust their prices ex–ante with $\pi^\star$, this result follows directly. Given $r^n = \bar{\rho}$ the higher $\pi^\star$ the higher will be the steady state nominal rates while at the same time a higher $\pi^\star$ is costless in terms of welfare as the nominal anchor grows along with $\pi^\star$. If, however, such an adjustment is not feasible, raising the inflation target imposes a trade–off which will be analyzed in future research.

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