Q-Targeting in New Keynesian Models*

Burkhard Heer[ab], Alfred Maußner[c]

*This work is supported from the German Science Association (Deutsche Forschungsgemeinschaft) under grant MA 1110/3-1 within its priority program "Financial Market Imperfections and Macroeconomic Performance". We gratefully acknowledge this support.

[a]University of Augsburg, Department of Economics, Universitätsstraße 16, 86159 Augsburg, Germany, burkhard.heer@wiwi.uni-augsburg.de

[b]CESIfo

[c]Corresponding author, University of Augsburg, Department of Economics, Universitätsstraße 16, 86159 Augsburg, Germany, alfred.maussner@wiwi.uni-augsburg.de

June 10, 2014

JEL classification: E12, E32, E52, G12

Key Words:

Asset Prices, Monetary Policy, New Keynesian Model, Q Targeting

Abstract

We consider optimal monetary policy in a model that integrates credit frictions in the standard New Keynesian model with sticky prices and wages as well as adjustment costs of capital. Different from traditional models with credit frictions such as Carlstrom and Fuerst (1998), the model is able to generate an anti-cyclical external finance premium as observed empirically in the US economy. Monetary policy is characterized by a Taylor rule according to which the nominal interest rate is set as a function of the deviation of the inflation rate from its target rate, the output gap, and Tobin's q. The latter is measured by the relative price of newly installed capital. We show that that monetary policy should optimally decrease interest rates with higher capital prices. However, the consideration of Tobin's q implies only small welfare effects.

1 Introduction

The financial crises of 2007 has triggered renewed interest into a debate which started in the late 1990: should central banks target asset prices? Bernanke and Gerlter (1999, 2001) were among first to ask how central bankers should react to asset price volatility. Bernanke and Gertler argue that there is no need for concern if asset price movements reflect changes in economic fundamentals. In this case, asset prices are only relevant with regard to monetary policy if they convey additional information about the state of the econommy. However, if asset prices were driven by nonfundamental factors, their influence could be destabilizing. For this case, they consider a bursting asset price bubble in a version of the model developed in Bernanke, Gertler, and Gilchrist (1999) and show that asset price targeting may even destabilize the economy.

More recently, the question of asset price targeting has been analyzed in models that introduce financial frictions in the standard New Keynesian business cycle. For instance, Carlstrom and Fuerst (2007) argue that asset price targeting may increase the parameter region within which the rational expectations equilibrium is not unique so that sunspot equilibria arise. Machado (2012) considers learning in the model of Carlstrom and Fuerst (2007) and shows that asset price targeting may hamper the convergence to the rational expectations equilibrium. Christiano et al. (2010) study a purely news-driven upswing in a model with the financial accelerator of Bernanke, Gertler, and Gilchrist (1999). The central bank can moderate the effects of this kind of shock if it also reacts to the increased credit demand of borrowers. Faia and Monacelli (2007) introduce a nominal price rigidity into the financial accelerator model of Carstrom and Fuerst (1997). The interaction between the nominal friction and the financial friction requires a negative response of the nominal interest rate set by the central bank and the relative price of capital. Moreover, the welfare gains of targeting the price of capital in addition to inflation are very small as compared to a strict anti-inflation policy. However, it is questionable whether this result holds in a more general setting because, in their model, business cycle fluctuations are mainly driven by shocks to total factor productivity. The present paper is intended to fill this gap.

In this paper, we follow Faia and Monacelli (2007) and consider the desirability of asset price targeting with respect to its effect on the welfare of the representative household. We employ the approach pioneered by Schmitt-Grohé and Uribe (2004, 2005, 2007) and compute the welfare effects of an extended Taylor rule relative to a simple Taylor rule that just reacts to the deviation of inflation from the central bank's target. As in these papers we disregard rules that i) lead to indeterminacy and ii) may hurt the zero lower bound. We consider different kinds of financial frictions and a richer structure of shocks. In this way we provide a more balanced view on the desirability of asset price targeting.

The paper is structured as follows. In the next section we introduce a first model with the usual shock to total factor productivity and a government spending shock. The model features two nominal frictions (price and wage staggering a la Calvo (1982)) and a financial friction in the production of primary goods as proposed by Carlstrom and Fuerst (1998). Section 3 presents the calibration of the model. In Section 4, we present our results. The conclusion of Faia and Monacelli (2007) remains intact in our model with sticky wages and multiple shocks: the welfare gains of targeting the relative price of capital are negligible. Section 5 concludes.

2 The Model

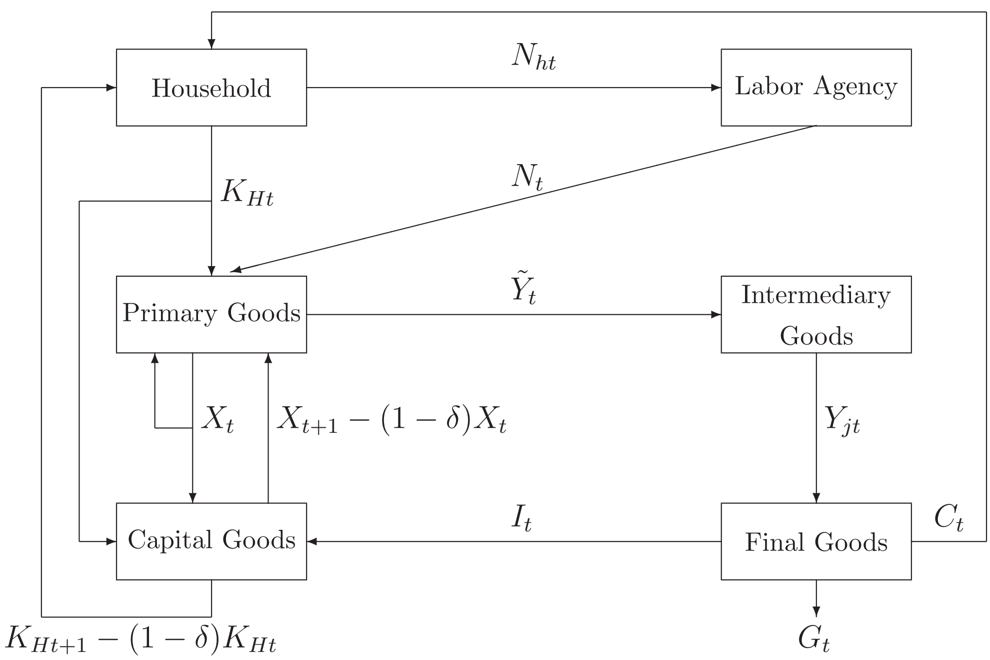
The model merges a standard New Keynesian model with sticky nominal prices and wages as, e.g., in Erceg, Henderson, and Levin (2000) and Christiano, Eichenbaum, and Evans (2005), and adjustment costs of capital as, e.g., in Jermann (1998) and Bernanke, Gertler, and Gilchrist (1999), with the credit friction model of Carlstrom and Fuerst (1998).

2.1 Structure of the Model

The model consists of a household, the government, a labor agency, a sector of primary goods producers, a wholesale sector, a final goods sector, and a capital goods producing sector. Time is discrete and denoted by t. Figure 2.1 illustrates the flows of factor services and goods between the household and the various sectors of the economy.

The household has a unit mass of members who rent their labor services N_{ht} to the labor agency. The agency sells a composite N_t of these services to the primary goods producers. Each firm $f \in [0,1]$ in this sector employs labor N_t and capital services from the household sector K_{Ht} and from other firms X_t to produce $\tilde{Y}_t$ units of a good, which serves as input in the production of intermediary goods. Each firm $j \in [0,1]$

Figure 2.1: Structure of the Model



in this sector produces a differentiated good Y_{jt} and sells it to the final goods sector. This sector bundles the intermediary goods and sells consumption goods C_t to the household, investment goods I_t to the capital goods sector and public goods G_t to the government. New capital goods are produced from capital services rented from the household and primary goods producers and from investment goods. They are sold to primary goods producers $X_{t+1} - (1-\delta)X_t$ and to the household $K_{Ht+1} - (1-\delta)K_{Ht}$.

2.2 Final Goods

The firm in this sector buys the brands Y_{jt}, $j \in [0, 1]$ at the nominal price P_{jt} from the intermediary goods sector and combines them to a final good Y_t, which is sold at the nominal price P_t to the household as consumption good C_t and to the capital goods production sector as investment good I_t. The technology is given by

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{\epsilon_y - 1}{\epsilon_y}} dj \right]^{\frac{\epsilon_y}{\epsilon_y - 1}}, \quad \epsilon_y > 1. \tag{2.1}$$

The zero-profit condition

$$P_t Y_t = \int_0^1 P_{jt} Y_{jt} dj$$

implies the usual demand function for intermediary good j:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\epsilon_y} Y_t, \tag{2.2}$$

where

$$P_t = \left(\int_0^1 P_{jt}^{1-\epsilon_y} dj\right)^{\frac{1}{1-\epsilon_y}}. \tag{2.3}$$

is the price index.

2.3 Capital Goods

We implement adjustment costs of capital as in Bernanke, Gertler, and Gilchrist (1999). New capital goods are produced from capital services $K_t = K_{Ht} + X_t$ rented at the price r_{Kt} and from investment goods I_t according to the function $\Psi(I_t/K_t)K_t$. They are sold at the price q_t. The function Ψ is increasing in its argument and strictly concave. As usual, we assume that it is costless to keep the capital stock constant: $\Psi(\delta) = \delta$ and $\Psi'(\delta) = 1$, where δ is the rate of capital depreciation. In our numerical simulations we employ the function

$$\Psi(I_t/K_t) = \frac{a_1}{1-\zeta}\left(\frac{I_t}{K_t}\right)^{1-\zeta} + a_2. \tag{2.4}$$

Profit maximization,

$$\max_{K_t, I_t} \quad q_t \Psi(I_t/K_t)K_t - r_{Kt}K_t - I_t$$

implies

$$q_t = \frac{1}{\Psi'(I_t/K_t)}, \tag{2.5a}$$

$$r_{Kt} = q_t \Psi(I_t/K_t) - (I_t/K_t) \tag{2.5b}$$

so that profits are zero in equilibrium.

2.4 Intermediary Goods and Price Setting

A firm $j \in [0, 1]$ in the intermediary sector buys goods at the nominal price P_{yt} from the primary production sector, brands it and sells it at the price P_{jt} to the final goods sector. Its profit in units of the final product equals

$$D_{jt} = \left(\frac{P_{jt}}{P_t} - g_t\right) Y_{jt}, \quad g_t = \frac{P_{yt}}{P_t} \tag{2.6}$$

and is distributed to the household sector. In each period t a randomly selected fraction $1 - \varphi_y$ of firms in this sector receives the signal to optimally choose their relative price $p_{At} = P_{At}/P_t$. The remaining fraction is allowed to raise their nominal price P_{Nt} according to the inflation rate observed in the previous period:

$$P_{Nt} = \pi_{t-1} P_{Nt-1}, \quad \pi_t = \frac{P_t}{P_{t-1}}. \tag{2.7}$$

2.5 Primary Production

Primary production is organized in a sector with a unit mass of firms $f \in [0, 1]$. In Carlstrom and Fuerst (1998) these firms are owned by risk-neutral entrepreneurs. We follow Chugh (2013) and assume that the household owns the firms but that firms are more impatient than the household. This reflects an un-modeled principal agent problem that drives a wedge between the interest of the household and the management of the firm. Its effect is to prevent full self-financing of firms.

Firm Assets. The firms need credit to pay for their factor services in advance. In order to get credit they have to accumulate assets. Let X_{ft} denote the stock of capital owned by firm f at the beginning of period t. The firm rents this capital at the price r_{Yt} to other primary goods producing firms. When production in this sector has taken place it rents the same amount to the capital goods sector at the rate r_{Kt}. In addition to its factor income the firm receives a small transfer Δ_{ft} from the household. This ensures that the firm will be able to continue its operates even in the case of credit default. The transfer is deducted from the firms dividend payment to the household. The net worth NW_{ft} of the firm, therefore, is equal to

$$NW_{ft} = (q_t(1 - \delta) + r_{Yt} + r_{Kt})X_{ft} + \Delta_{ft}. \tag{2.8}$$

Production and Factor Demand. The firm f employs labor N_{ft} and capital K_{ft} to produce the amount

$$\tilde{Y}_{ft} = \omega_{ft} Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha}, \quad \alpha \in (0, 1). \tag{2.9}$$

ω_{ft} is an idiosyncratic shock, distributed iid with density ϕ and mean $\mathbb{E}(\omega_{ft}) = \Omega_t$. Z_t is an aggregate shock that is governed by

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \sigma_Z \epsilon_t, \quad \rho_Z \in [0, 1), \ \epsilon_t \sim \mathcal{N}(0, 1). \tag{2.10}$$

The firm must pay for its factor services

$$M_{ft} = w_t N_{ft} + r_{Yt} K_{ft} \tag{2.11}$$

in advance, where w_t is the wage rate in units of the final good. The firm observes Z_t but not ω_{ft} before it decides on the size of its credit $M_{ft} - NW_{ft}$. After it has observed ω_{ft} the firm maximizes

$$g_t \omega_{ft} Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha}$$

subject to $M_{ft} \geq w_t N_{ft} + r_{Yt} K_{ft}$. The first-order conditions

$$\begin{aligned} \lambda_{ft} w_t &= (1-\alpha) g_t \omega_{ft} Z_t N_{ft}^{-\alpha} K_{ft}^{\alpha}, \\ \lambda_{ft} r_{Yt} &= \alpha g_t \omega_{ft} Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha-1}, \end{aligned}$$

imply $w_t / r_{Yt} = ((1-\alpha)/\alpha)(K_{ft}/N_{ft})$ so that all firms employ the same capital-labor-ratio $k_t = (K_{ft}/N_{ft})$. As a consequence, the scaled Lagrange multiplier of the constraint $\lambda_{ft}/\omega_{ft} \equiv v_t$ is independent of the firm index f, and

$$v_t M_{ft} = g_t Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha} = g_t Z_t k_t^{\alpha} N_{ft}. \tag{2.12}$$

Thus, in terms of the final good, v_t is a mark-up on factor costs M_{ft}. For later reference note that this relation can be integrated to

$$v_t M_t = g_t Z_t k_t^{\alpha} N_t, \tag{2.13}$$

where $x_t = \int_0^1 x_{ft} df$ for $x \in \{M, N, K\}$ and that the first-order conditions for factor demand can be written in terms of aggregate variables:

$$w_t = (1-\alpha)(g_t/v_t)\tilde{Y}_t/N_t, \tag{2.14a}$$

$$r_{Yt} = \alpha (g_t/v_t)\tilde{Y}_t/K_t, \tag{2.14b}$$

$$\tilde{Y}_t = Z_t N_t^{1-\alpha} K_t^{\alpha}. \tag{2.14c}$$

The Credit Contract. The firm borrows the amount $M_{ft} - NW_{ft}$ intra-period from the household. The realization of ω_{ft} is private information. If the creditor wishes to see the firm's production, he must pay a screening cost. This cost is proportional to factor costs in terms of the final good $v_t M_{ft}$ with factor of proportionality κ. This is the costly state verification framework of Townsend (1979), Gale and Hellwig (1985), and Williamson (1987) employed in Carlstrom and Fuerst (1997, 1998).

The credit contract specifies M_{ft}, the lending rate r_{Lt}, and a bankruptcy threshold $\bar{\omega}_{ft}$, given by

$$\bar{\omega}_{ft} = (1 + r_{Lt}) \frac{M_{ft} - NW_{ft}}{g_t Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha}}, \tag{2.15}$$

so that for $\omega_{ft} < \bar{\omega}_{ft}$ the firm defaults and the creditor seizes the firm's output less the screening costs. Otherwise the firm redeems the loan, pays the interest and keeps all of its production. Because the household lends to all firms, he can fully diversify the risk and acts as if he was risk averse. The expected return for the firm equals

$$\int_{\bar{\omega}_{ft}} \omega_{ft} g_t Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha} \phi(\omega_{ft} d\omega_{ft}) - (1 - \Phi(\bar{\omega}_{ft}))(1 + r_{Lt})(M_{ft} - NW_{ft}),$$

$$\Phi(\bar{\omega}_{ft}) = \int^{\bar{\omega}_{ft}} \phi(\omega_{ft}) d\omega_{ft}.$$

Using (2.15) this can be written as $g_t Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha} f(\bar{\omega})$, where

$$f(\bar{\omega}) = \int_{\bar{\omega}_t} \omega_t \phi(\omega_t) d\omega_t - (1 - \Phi(\bar{\omega}_t))\bar{\omega}_t. \tag{2.16}$$

Note that from (2.12) the expected return to the borrower can also be written as a fraction of the factor costs in terms of final output, $v_t M_{ft} f(\bar{\omega}_t)$. Note also that $f(\cdot)$ depends on the distribution ϕ so that $\bar{\omega}_t$ is independent of the firm index (which we therefore have dropped). The expected return of the creditor equals

$$\int^{\bar{\omega}_t} \omega_t g_t Z_t N_{ft}^{1-\alpha} K_{ft}^{\alpha} \phi(\omega_t d\omega_t) + (1 - \Phi(\bar{\omega}_{ft}))(1 + r_{Lt})(M_{ft} - NW_{ft}) - \Phi(\bar{\omega}_t)\kappa v_t M_{ft}.$$

Using (2.15) and (2.14c) this is equal to $v_t M_{ft} g(\bar{\omega}_t)$ with

$$g(\bar{\omega}_t) = \int^{\bar{\omega}_t} \omega_t \phi(\omega_t) d\omega_t + (1 - \Phi(\bar{\omega}_t))\bar{\omega}_t - \Phi(\bar{\omega}_t)\kappa. \tag{2.17}$$

Finally, note that

$$f(\bar{\omega}_t) + g(\bar{\omega}_t) = \Omega_t - \Phi(\bar{\omega}_t)\kappa. \tag{2.18}$$

The optimal pair $(M_{ft}, \bar{\omega}_t)$ maximizes the expected return of the firm $v_t M_{ft} f(\bar{\omega}_t)$ subject to the participation constraint of the household. Because the loan is intra-period the household will be indifferent between lending to a producer or keeping his funds if he will at least get back his loan: $v_t M_{ft} g(\bar{\omega}_t) \geq M_{ft} - NW_{ft}$. $v_t M_{ft} g(\bar{\omega}_t) \geq M_{ft} - NW_{ft}$. This pair solves

$$1 = v_t \left[\Omega_t - \Phi(\bar{\omega}_t)\kappa - \frac{f(\bar{\omega}_t)\phi(\bar{\omega}_t)\kappa}{1 - \Phi(\bar{\omega}_t)} \right], \tag{2.19a}$$

$$M_{ft} = \frac{NW_{ft}}{1 - \upsilon_t g(\bar{\omega}_t)}. \tag{2.19b}$$

Note that condition (2.19a) determines the bankruptcy threshold as a function of the markup on factor costs υ_t, which, thus, is equal for all firms f. Also note that equations (2.19b) and (2.8) can be aggregated over all firms in the primary production sector.

Asset Accumulation of the Firm. We assume that the firm distributes

$$D_{ft} = \upsilon_t M_{ft} f(\bar{\omega}_t) - \Delta_{ft} - q_t X_{ft+1} \tag{2.20}$$

as dividends to the household. As we shall demonstrate in a moment, the household's discount factor for returns from period $t+s$ is equal to $\beta^s \Lambda_{t+s}/\Lambda_t$, where Λ_t is the multiplier of the household's budget constraint. The firm is more impatient than the household and employs the discount factor $(\beta\gamma)^s \Lambda_{t+s}/\Lambda_t$ with $\gamma \in (0,1)$. Therefore, the value of the firm is given by

$$V_{ft} = \mathbb{E}_t \sum_{s=0}^{\infty} (\gamma\beta)^s \frac{\Lambda_{t+s}}{\Lambda_t} D_{ft+s}.$$

Substituting for D_{ft} from (2.20), for M_{ft} from (2.19b), and for NW_{ft} from (2.8) and maximizing with respect to X_{ft+1} yields the Euler equation

$$q_t = \gamma\beta \frac{\Lambda_{t+1}}{\Lambda_t} \left[q_{t+1}(1-\delta) + r_{Yt+1} + r_{Kt+1} \right] \frac{\upsilon_{t+1} f(\bar{\omega}_{t+1})}{1 - \upsilon_{t+1} g(\bar{\omega}_{t+1})}. \tag{2.21}$$

2.6 Labor Demand

The household has a unit mass of members $h \in [0,1]$ who sell their labor services N_{ht} at the wage rate W_{ht} to an agency. The agency bundles them into a single service,

$$N_t = \left[\int_0^1 N_{ht}^{\frac{\epsilon_n - 1}{\epsilon_n}} dh \right]^{\frac{\epsilon_n}{\epsilon_n - 1}}, \quad \epsilon_n > 1, \tag{2.22}$$

and sells this service at the nominal wage W_t to the primary good producers. The zero profit condition

$$W_t N_t = \left(\int_0^1 W_{ht} N_{ht} dh \right)$$

implies the demand function

$$N_{ht} = \left(\frac{W_{ht}}{W_t} \right)^{-\epsilon_n} N_t, \tag{2.23}$$

and the wage index

$$W_t = \left[\int_0^1 W_{ht}^{1-\epsilon_n} \right]^{\frac{1}{1-\epsilon_n}}. \tag{2.24}$$

2.7 Wage Setting

The current period utility u of household member h depends on his consumption C_{ht}, labor supply N_{ht} and the consumption habit $\bar{C}_t$. We parameterize u as:

$$u(C_{ht}, N_{ht}, \bar{C}_t) = \frac{(C_{ht} - \chi \bar{C}_t)^{1-\eta} - 1}{1-\eta} - \frac{\nu_0}{1+\nu_1} N_{ht}^{1+\nu_1}, \quad \eta, \nu_0, \nu_1 \geq 0, \ \chi \in [0, 1). \tag{2.25}$$

In equilibrium $\bar{C}_t$ equals previous period's aggregate consumption $C_{t-1} = \int_0^1 C_{ht-1} dh$.

In each period a random fraction $1 - \varphi_n$ of the household members receive a signal to choose their nominal wage W_{At} optimally. The remaining fraction φ_n is allowed to increase their wage W_{Nt} according to the price inflation observed in the past period:

$$W_{Nt} = a\pi_{t-1} W_{Nt-1}, \quad \pi_t = \frac{P_t}{P_{t-1}}. \tag{2.26}$$

Those who optimize set their real wage $\tilde{w}_t = W_{At}/P_t$ to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta \varphi_n)^s u(C_{ht+s}, N_{ht+s}, \bar{C}_{t+s})$$

subject to labor demand (2.23) and the budget constrain

$$\frac{W_{ht}}{P_t} N_{ht} + RMT_t \geq 0,$$

where RMT_t is a stand in for the remaining terms of this constraint to be introduced in the next subsection.

2.8 Consumption and Portfolio Allocation

As usual we assume that the members of the household pool their income so that their decision to consume and save is subject to a budget constraint in which we can ignore the index h. The representative household owns two different kinds of assets:[1] physical

[1] In addition, he lends intra-period to firms in the primary sector. Since – as noted above – he receives his loan back at the end of the period, we ignore the loan in the budget constraint.

capital K_{Ht} and nominal bonds B_t. The latter pay the predetermined nominal interest rate $Q_t - 1$. The former yield a factor income of $(r_{Yt} + r_{Kt})K_{Ht}$ because capital is first employed in the production of primary goods and then in production of capital goods.[2] In addition to interest income, rental income, and wage income $w_t N_t$, the household receives dividends from the primary goods producers $\int D_{ft}df$ and dividends from the intermediary goods producers $\int D_{jt}dj$. He pays taxes T_t to the government, and spends the remaining income on consumption C_t and asset accumulation. His budget constraint in terms of the final goods, therefore, reads:

$$\begin{aligned} &w_t N_t + (r_{Yt} + r_{Kt})K_{Ht} + \int_0^1 D_{jt}dj + \int_0^1 D_{ft}df + (Q_t - 1)\frac{B_t}{P_t} - T_t \\ &\geq C_t + q_t(K_{Ht+1} - (1-\delta)K_{Ht}) + \frac{B_{t+1} - B_t}{P_t}. \end{aligned} \tag{2.27}$$

Per capita consumption $C_{ht} = C_t$ and the future stock of capital K_{Ht+1} are determined from maximizing

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \int_0^1 u(C_{ht+s}, N_{ht+s}, \bar{C}_{t+s})dh$$

subject to the budget constraint (2.27). The respective first-order conditions are

$$\Lambda_t = (C_t - \chi \bar{C}_t)^{-\eta}, \tag{2.28a}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\left(q_{t+1}(1-\delta) + r_{Yt+1} + r_{Kt+1}\right), \tag{2.28b}$$

$$1 = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\frac{Q_{t+1}}{\pi_{t+1}}. \tag{2.28c}$$

2.9 Government

The government's budget constraint is

$$\frac{B_{t+1} - B_t}{P_t} + T_t = (Q_t - 1)\frac{B_t}{P_t} + G_t. \tag{2.29}$$

We assume $B_t = 0$ for all t and that government spending G_t is governed by

$$\ln G_t = (1-\rho_G)\ln G + \rho_G \ln G_{t-1} + \sigma_G \epsilon_t, \quad \rho_G \in [0,1),\ \epsilon_t \sim \mathcal{N}(0,1). \tag{2.30}$$

[2] Note the slight abuse of notation: K_{ht} refers to the total stock of capital owned by the household and not to capital owned by household member h.

2.10 Monetary Authority

The central bank sets the nominal interest rate Q_{t+1} according a Taylor rule. The rule includes the previously set interest rate Q_t to account for sluggish adjustment, the deviation of the inflation π_t rate from the target rate π, the deviation of Tobin's q q_t from its steady state value of $q = 1$, and the deviation of output Y_t from its stationary level Y:

$$Q_{t+1} = Q_t^{\delta_1} \left(\frac{\pi}{\beta}\right)^{1-\delta_1} \left(\frac{\pi_t}{\pi}\right)^{\delta_2} (q_t)^{\delta_3} (Y_t/Y)^{\delta_4}, \quad \delta_1 \in [0, 1). \tag{2.31}$$

The choice of the parameters δ_2, δ_3, and δ_4 must satisfy two requirements: (i) the equilibrium dynamics of the economy must be determinate and (ii) the Taylor rule is subject to the zero lower bound, (i.e. $Q_t \geq 1$.)

2.11 Equilibrium Dynamics

In equilibrium all markets clear. Capital services employed in the production of primary goods equal

$$K_t = K_{Ht} + X_t, \quad X_t = \int_0^1 X_{ft} df \tag{2.32}$$

and accumulate according to

$$K_{t+1} - (1 - \delta)K_t = \Psi(I_t/K_t)K_t. \tag{2.33}$$

Equation (2.20) implies

$$q_t X_{t+1} = f(\bar{\omega}_t) g_t \tilde{Y}_t - \int_0^1 (D_{ft} - \Delta_{ft}) df, \tag{2.34}$$

where the right-hand side of equation (2.13) was used to replace $v_t M_t$. Aggregating equation (2.8) over all primary production firms yields

$$NW_t = [q_t(1 - \delta) + r_{Yt} + r_{Kt}] X_t + \int_0^1 \Delta_{ft} df. \tag{2.35}$$

Condition (2.19b) and equation (2.13) imply

$$\tilde{Y}_t = \frac{NW_t}{1 - v_t g(\bar{\omega}_t)}. \tag{2.36}$$

Consolidating the budget constraints of the household, the government, and the definition of dividend payments to the household yields

$$g_t \tilde{Y}_t(\Omega_t - \Phi(\bar{\omega}_t)\kappa) + \int_0^1 \left(\frac{P_{jt}}{P_t} - g_t \right) Y_{jt} dj = C_t + I_t + G_t.$$

Market clearing for intermediary goods requires $\int_0^1 Y_{jt} dj = \tilde{Y}_t$ and the first part of the intergral term equals Y_t (see (2.2)). Hence, the preceding equation reduces to the resource constraint:

$$Y_t + g_t \tilde{Y}_t(\Omega_t - 1 - \Phi(\bar{\omega}_t)\kappa) = C_t + I_t + G_t. \tag{2.37}$$

This may look somewhat uncommon. But note that Ω_t acts as a second shock to aggregate factor productivity in the primary goods production. For $\Omega_t = 1$ (as assumed by Carlstrom and Fuerst (1997, 1997)) the left-hand side reduces to $Y_t - g_t \tilde{Y}_t \Phi(\bar{\omega}_t)\kappa$. The second term are the resources employed in screening bankrupt firms in the primary production sector in units of the final good. Without those costs we are back at the usual constraint $Y_t = C_t + I_t + G_t$.

We present the full system of equations that determine the dynamics of the model in the Appendix.

2.12 Welfare Analysis

Our goal is to determine whether or not the inclusion of Tobin's q in the Taylor rule (2.31) does improve monetary policy. Our point of reference is the welfare associated with $\delta_1 = \delta_3 = \delta_4 = 0$ and $\delta_2 = 1.5$. Let

$$\begin{aligned} V_t &= V_t^C - V_t^N, \\ V_t^C &= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{(C_{t+s} - \chi C_{t+s-1})^{1-\eta} - 1}{1-\eta} \right], \\ V_t^N &= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{\nu_0}{1+\nu_1} \tilde{N}_{t+s}^{1+\nu_1} \right], \end{aligned} \tag{2.38}$$

denote the expected discounted life-time utility of the family associated with this solution, where $\tilde{N}_t^{1+\nu_1} = \int_0^1 N_{ht}^{1+\nu_1} dh$.

We solve the model for non-zero coefficients δ_i, $i = 1, \ldots, 4$ on a four-dimensional grid.

Let

$$\begin{aligned}\bar{V}_t &= \bar{V}_t^C - \bar{V}_t^N,\\ \bar{V}_t^C &= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{(\bar{C}_{t+s} - \chi \bar{C}_{t+s-1})^{1-\eta} - 1}{1-\eta} \right],\\ \bar{V}_t^N &= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{\nu_0}{1+\nu_1} \tilde{\bar{N}}_{t+s}^{1+\nu_1} \right].\end{aligned} \tag{2.39}$$

be life-time utility implied by any of these solutions. We determine λ so that[3]

$$\bar{V}_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{((1-\lambda)C_{t+s} - \chi(1-\lambda)C_{t+s-1})^{1-\eta} - 1}{1-\eta} - \frac{\nu_0}{1+\nu_1} \tilde{N}_{t+s}^{1+\nu_1} \right] \tag{2.40}$$

Like the policy functions that solve the model, λ is a function of the given initial state of the system. In our model the vector of state variables $\mathbf{x}$ includes the aggregate stock of capital K_t, the nominal interest rate factor Q_t, aggregate consumption and the real wage as of period $t-1$, C_{t-1}, and w_{t-1}, the lagged rates of price and wage inflation, π_{t-1}, and ω_{t-1}, the measures of price and wage dispersion s_{t-1}^y and s_{t-1}^n, respectively,[4], the log of total factor productivity $\ln Z_t$, and the shock in the Taylor rule $\sigma^Q \epsilon_t$. We approximate $\lambda(\mathbf{x})$ at the point $\mathbf{x} = [K, Q, C, w, \pi, \omega, 1, 0, 0]$, where the K, $Q = \pi/\beta$, C, w, π, ω denote, respectively, the capital stock, the interest rate factor, consumption, the real wage, price and wage inflation on the balanced growth path of the deterministic counterpart of the model. In the Appendix we show that up to second order accuracy

$$\lambda = \frac{1-\beta}{1+(1-\eta)(1-\beta)V^C} \left[V_{\sigma\sigma}^C + \bar{V}_{\sigma\sigma}^N - \bar{V}_{\sigma\sigma}^C - V_{\sigma\sigma}^N \right]. \tag{2.41}$$

In this expression, $V_{\sigma\sigma}^i$ are the second partial derivatives of V^i, $i = C, N$ with respect to the scaling parameter in the driving process of the shocks, $\mathbf{z}_t = \Pi \mathbf{z}_{t-1} + \sigma \Omega \boldsymbol{\epsilon}_t$,[5] $\mathbf{z}_t = [\ln Z_t]$.

3 Calibration

We calibrate the model with respect to the U.S. economy. The length of the period is one quarter. Table 3.1 summarizes the model's parameters and the values assigned to

[3] Schmitt-Grohé and Uribe (2004b) do not compensate for consumption at time $t-1$. Their definition yields a smaller welfare measure since the household's utility is a decreasing function of the habit. However, because the ranking of different monetary policy rules is independent of the scale of the welfare measure, we use the analytically more convenient definition.

[4] See the Appendix for the definition of these variables.

[5] See Schmitt-Grohé and Uribe (2004) for this representation.

them.

As far as possible we follow Christiano, Eichenbaum, and Evans (2005). Accordingly,

- we choose a steady state real interest rate of 3 percent per annum so that $\beta = 1.03^{-0.25}$,
- employ log-preferences with respect to consumption so that $\eta = 1$,
- set the habit parameter χ equal to 0.65,
- assume a Frisch elasticity of labor supply $1/\nu_1$ equal to unity,
- determine the parameter ν_0 so that the steady state value of N equals one,
- set the capital share in output equal to $\alpha = 0.36$,
- the rate of capital depreciation equal to $\delta = 0.025$,
- the price elasticity of demand for intermediary goods equal to $\epsilon_y = 6$,
- the wage elasticity of labor demand equal to $\epsilon_n = 21$,
- the fraction of firms not setting their price optimally equal to $\phi_y = 0.60$, and
- the fraction of household members not setting their wage optimally equal to $\phi_n = 0.64$.

With respect to the credit friction we draw on Carlstrom and Fuerst (1997). They employ $\kappa = 0.25$ for the costs of bankruptcy, use a log-normal distribution of the idiosyncratic shock ω, and determine the parameters of this distribution as well as the bankruptcy threshold $\bar{\omega}$ from three targets: a mean of one, a quarterly bankruptcy rate of 0.974 percent, and an annual external finance premium of 187 basis points. Given $\bar{\omega}$, equation (2.19a) determines the mark-up υ, and he value of the additional discount factor γ follows from the steady-state versions of equations (2.21) and (2.28b) as:

$$\gamma = \frac{1 - \upsilon g(\bar{\omega})}{\upsilon f(\bar{\omega})}.$$

The steady state share of government spending in output $G/Y = 0.16$ as well as the parameters of the TFP shock and the government spending shock stem from Schmitt-Grohé and Uribe (2007). We also follow Schmitt-Grohé and Uribe (2005) and set the

Table 3.1

Parameter	Value	Description
β	$1.03^{-0.25}$	Subjective discount factor
$1/\eta$	1	Intertemporal elasticity of substitution
$1/\nu_1$	1	Frisch elasticity of labor supply
χ	0.65	Habit parameter
N	1	Steady state labor supply
α	0.36	Share of capital in value added
δ	0.025	Rate of capital depreciation
ζ	$\{0.5, 2.5\}$	Elasticity of marginal adjustment cost function Ψ'
ρ_Z	0.856	Autocorrelation of TFP shock
σ_Z	0.0064	Standard deviation of innovations of TFP shock
$E(\omega)$	1	Mean of distribution of idiosyncratic productivity shock
κ	0.25	Costs of bankruptcy
$\Phi(\bar{\omega})$	0.00974	Steady state bankruptcy rate
$1 + r_L$	$1.0187^{0.25}$	Gross external finance premium
ϵ_y	6	Price elasticity of demand for intermediary goods
ϵ_n	21	Wage elasticity of labor demand
ϕ_y	0.60	Fraction of intermediary goods firms not setting their prices optimally
ϕ_n	0.64	Fraction of household members not setting their wages optimally
G/Y	0.16	Share of government spending in steady state production
ρ_G	0.87	Autocorrelation parameter in government spending shock
σ_G	0.016	Standard deviation of innovations in government spending shock
π	$1.042^{0.25}$	Steady state inflation factor

steady-state inflation rate equal to the average growth rate of the U.S. GDP deflator over the period 1960-1998, which gives $\pi = 1.042^{0.25}$.

Finally, to consider the potential of our model to produce a counter-cyclical external finance premium, we disregard the spillover from the aggregate shock to the mean of the distribution of the idiosyncratic productivity modeled in Faia and Monacelli (2007). Therefore, $\Omega_t = 1$ for all periods.

4 Results

In this section, we present our results how the introduction of a q-target in the Taylor rule affects the utility of the households. In particular, we search for the optimal monetary policy rule and analyse if monetary policy should respond to higher asset prices by lowering or increasing interest rates. Our benchmark is the Taylor rule (2.31) with zero coefficients on the past interest rate $\delta_1 = 0$, a coefficient of $\delta_2 = 1.5$ on the inflation gap, and zero coefficients on capital price $\delta_3 = 0$ and the output gap $\delta_4 = 0$. We compute the welfare gains of policies with non-zero δ_i, $i = 1, 2, 3, 4$ over the grid

$$\begin{aligned} \delta_1 &\in [0, 0.95], \\ \delta_2 &\in [1.2, 2.5], \\ \delta_3 &\in [-2.5, 2.5], \\ \delta_4 &\in [0, 2.5] \end{aligned}$$

for two different values of the parameter ζ, indicating small and medium size costs of capital accumulation.

Table 4.1 presents the results obtained for the benchmark model without the financial friction. Apart from the monopoly power in product and labor markets, this model embeds three kind of distortions: 1) The variable mark-up (the inverse of the variable g_t) over marginal costs and the variable mark-up over the marginal rate of substitution between leisure and consumption introduces inefficient fluctuations of hours and production. 2) The price and 3) wage dispersion forces the household members to spread consumption and labor supply unevenly over the continuum of consumption goods and labor services, respectively.[6] The Taylor rule that maximizes the welfare gain of the

[6]The latter two effects are not present in the model of Faia and Monacelli (2007), because they assume convex costs of price adjustment so that, in the symmetric equilibrium of the product market, all firms will choose the same price. In addition, they do not model sticky wages.

Table 4.1

Welfare Effects: Benchmark Model Without Financial Friction

	$\zeta = 0.5$		$\zeta = 2.5$	
	ii	iii	iv	v
δ_1	0.78	0.0	0.47	0.0
δ_2	2.38	2.5	1.20	2.5
δ_3	−1.41	0.0	−0.46	0.0
δ_4	1.79	0.75	0.75	0.75
λ	−0.0628	−0.0451	−0.0341	−0.0240

Notes: ζ is the elasticity of Tobin's q with respect to the investment-capital ratio I/K. δ_i, $i = 1, 2, 3, 4$ denote the coefficients of the Taylor rule (2.31) on the past interest rate, the inflation gap, the price of capital, and the output gap, respectively. λ is the percentage of consumption that must be given (taken if positve) to the household in the pure inflation target regime with $\delta_2 = 1.5$ and $\delta_i = 0$, $i = 1, 3, 4$, to make him equally well-off as under the rule specified in columns ii-v.

household places a negative coefficient on the price of capital, $\delta_3 = -1.41$ ($\delta_3 = -0.46$) for the case of high (low) adjustment cost, $\zeta = 5.0$ ($\zeta = 2.5$). Compared to a policy which ignores this variable (columns iii and v), there is a welfare loss of about 0.018 ($\zeta = 0.5$) and 0.01 ($\zeta = 2.5$) percentage points, respectively.

The intuition behind this result rests on the observation that the cycle is mainly driven by the supply shock. A positive supply shock increases labor productivity. Due to the nominal frictions the real wage does not fully reflect this effect, the wedge between the marginal rate of substitution (MRS) and labor productivity widens. The household's supply of labor declines. If the central bank also reacts to the increased price of capital and lowers the nominal interest rate the household shifts consumption to the current period and the pressure on prices to fall is reduced. The gap between productivity and real wages diminishes and dampens or even overturns the negative effect on labor supply.

Table 4.2 presents the results for the benchmark model with the financial friction.[7] of the dynamic system of equations, some of which require numeric integration. In accordance with Faia and Monacelli (2007), we find that the optimal policy should

[7]Since the search for the optimal policy is relatively time-consuming in this model, we have not computed the welfare for policies that neglect the price of capital. The low speed of computation is caused by the repeated numerical evaluation of the Hessian matrix

Table 4.2
Welfare Effects: Benchmark Model With Financial Friction

	$\zeta = 0.5$	$\zeta = 2.5$
δ_1	0.36	0.52
δ_2	1.79	1.22
δ_3	−0.81	−0.48
δ_4	0.97	0.82
λ	−0.0617	−0.0355

Notes: See Table 4.1.

place again a negative weight on the price of capital. The welfare effects are as small as for the model without the financial friction.

There are two possible interpretations for the small quantitative effects: either the welfare effect of the financial friction are small and/or asset price targeting is not able to moderate this friction. In order to study the first explanation, we compare the welfare in the economy with and without frictions. For this reason, we simulate 1,000,000 time series of 1,000 periods for the two economies and compare the average utility of the households. We find that the average utility is smaller in the model without frictions and that the effect amounts to x% of total consumption. Accordingly, our results suggest that the introduction of a q-target rule does not help to moderate frictions in financial markets.

5 Conclusion

As our main result, we find that the welfare effects of monetary policy from targeting Tobin's are of negligible magnitude if business cycles are (mainly) driven by technology shocks. In this case, Tobin's q does not contain much additional information for the monetary authority that is not already reflected in the output gap and the inflation rate. However, we would like to caution the reader to interpret our results carefully because we have restricted our attention to the study of technology shocks. As pointed out by Christiano, Trabandt, and Walentin (2011), a shock to financial wealth is important for the explanation of business cycle fluctuations of GDP and investment. Following these authors, we are planning to introduce two additional kind of shocks in our model, a shock on financial wealth of entrepreneurs and a shock on consumption preferences. We expect that, as a consequence of these shocks (especially with respect to the first shock), the additional informational content of Tobin's q increases and, therefore, potential welfare effects will be magnified.

References

Bernanke, Ben S., and Mark Gertler. 1999. Monetary policy and asset price volatility. *Economic Review, Federal Reserve Bank of Kansas City*. Quarter IV. pp 17-51.

Bernanke, Ben S., and Mark Gertler. 2001. Should Central Banks Respond to Movements in Asset Prices? *American Economic Review*. Vol. 91(2). p. 253-257.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. 1999. The Financial Accelerator in a Quantitative Business Cycle Framework. in: John B. Taylor and Michael Woodford (eds.), Handbook of Macroeconomics, Volume 1C. pp. 1341-1393.

Carlstrom, Charles T. and Timothy S. Fuerst. 1997. Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis. *American Economic Review*. Vol. 87. pp. 893-910.

Carlstrom, Charles T. and Timothy S. Fuerst. 1998. Agency Costs and Business Cycles. *Economic Theory*. Vol. 12. pp. 583-597.

Christiano, Lawrence J. , Mathias Trabandt, and Karl Walentin. 2011. Introducing financial frictions and unemployment into a small open economy model. *Journal of Economic Dynamics and Control*. Vol. 35(12). pp. 1999-204.

Christiano, Lawrence J. , Martin Eichenbaum, and Charles L. Evans. 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*. Vol. 113. pp. 1-45.

Chugh, Sanjay K. 2013. Costly External Finance and Labor Market Dynamics. *Journal of Economic Dynamics and Control*. Vol. 37. pp. 2882-2912.

Erceg, Christopher J., Henderson, Dale W., Levin, Andrew D. 2000. Optimal Monetary Policy with Staggered Wage and Price Contracts. *Journal of Monetary Economics*. Vol. 46. pp. 281-313.

Gale, D. and M. Hellwig. 1985. Incentive-Compatible Debt Contracts. I. The One-Period Problem. *Review of Economic Studies*. Vol. 52. pp. 647-664

Heer, Burkhard and Alfred Maußner. 2013. Asset Returns, the Business Cycle, and the Labor Market. *German Economic Review*. Vol. 14(3). pp. 372-397.

Lütkepohl, Helmut. 2005. New Introduction to Multiple Time Series Analysis. Springer. Berlin.

Machado, Vicente da Gama. 2012. Monetary Policy and Asset Price Volatility. *Working Paper Series, Banco Central do Brasil*. No. 274.

Schmitt-Grohé, Stephanie and Martin Uribe. 2004. Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function. *Journal of Economic Dynamics and Control*. Vol. 28. pp. 755-775.

Schmitt-Grohé, Stephanie and Martin Uribe. 2005. Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model: Expanded Version. *National Bureau of Economic Research (NBER) Working Paper* No. W11417.

Schmitt-Grohé, Stephanie and Martin Uribe. 2007. Optimal Simple and Implementable Monetary and Fiscal Rules. *Journal of Monetary Economics*. Vol. 54. pp. 1702-1725.

Townsend, R.M. 1979. Optimal Contracts and Competitive Markets With Costly State Verification. *Journal of Economic Theory*. Vol. 21. pp. 265-293.

Williamson, S. 1987. Costly Monitoring, Optimal Contracts, and Equilibrium Credit Rationing. *Quarterly Journal of Economics*. Vol. 102. pp. 135-145.

Appendix

A Analysis of the Model

A.1 Price Setting

Consider the relative price P_{jt+s}/P_{t+s} of an intermediary goods producer j receiving the signal to choose its optimal relative price $p_{At} = P_{At}/P_t$ in period t and that has not been able to reset its price up to period $t+s$:

$$\frac{P_{jt+s}}{P_{t+s}} = \frac{\pi_{t+s-1}\cdots\pi_t}{\pi_{t+s}\cdots\pi_{t+1}} p_{At} = \frac{\pi_t}{\pi_{t+s}} p_{At}.$$

Accordingly, the firm will choose p_{At} in period t to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_y)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[\left(\frac{\pi_t}{\pi_{t+s}} p_{At}\right)^{-\epsilon_y} Y_{t+s} - g_{t+s} \left(\frac{\pi_t}{\pi_{t+s}} p_{At}\right)^{1-\epsilon_y} Y_{t+s} \right].$$

The first-order condition for this problem is:

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_y)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[(1-\epsilon_y) \left(\frac{\pi_t}{\pi_{t+s}}\right)^{1-\epsilon_y} Y_{t+s} p_{At}^{-\epsilon_y} + \epsilon_y g_{t+s} \left(\frac{\pi_t}{\pi_{t+s}}\right)^{-\epsilon_y} Y_{t+s} p_{At}^{-\epsilon_y - 1} \right]$$

and can be written as

$$p_{At} = \frac{\epsilon_y}{\epsilon_y - 1} \frac{\Gamma_{1t}}{\pi_t \Gamma_{2t}}, \tag{A.1a}$$

$$\Gamma_{1t} = \sum_{s=0}^{\infty} (\beta\varphi_y)^s \pi_{t+s}^{\epsilon_y} g_{t+s} \Lambda_{t+s} Y_{t+s} = \pi_t^{\epsilon_y} g_t \Lambda_t Y_t + (\beta\varphi_y) \mathbb{E}_t \Gamma_{1t+1}, \tag{A.1b}$$

$$\Gamma_{2t} = \sum_{s=0}^{\infty} (\beta\varphi_y)^s \pi_{t+s}^{\epsilon_y - 1} \Lambda_{t+s} Y_{t+s} = \pi_t^{\epsilon_y - 1} \Lambda_t Y_t + (\beta\varphi_y) \mathbb{E}_t \Gamma_{2t+1}. \tag{A.1c}$$

The price index (2.3) implies

$$P_t^{1-\epsilon_y} = (1-\varphi_y) P_{At}^{1-\epsilon_y} + \varphi_y P_{Nt}^{1-\epsilon_y} = (1-\varphi_y) P_{At}^{1-\epsilon_y} + \varphi_y (\pi_{t-1} P_{t-1})^{1-\epsilon_y}.$$

The second equality follows from the updating rule (2.7) and the fact that the non-optimizers are a random sample of optimizers and non-optimizers. Dividing by P_t on both sides delivers:

$$1 = (1-\varphi_y) p_{At}^{1-\epsilon_y} + \varphi_y (\pi_{t-1}/\pi_t)^{1-\epsilon_y}. \tag{A.1d}$$

Market clearing requires

$$\tilde{Y}_t = \int_0^1 Y_{jt} dj = \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\epsilon_y} Y_t dj = \underbrace{\left(\frac{\tilde{P}_t}{P_t}\right)^{-\epsilon_y}}_{\equiv s_t^y} Y_t, \quad \tilde{P}_t^{-\epsilon_t} \equiv \int_0^1 P_{jt}^{-\epsilon_y} dj,$$

so that

$$s_t^y Y_t = \tilde{Y}_t. \tag{A.1e}$$

Using the same reasoning for $\tilde{P}_t$ as for the price index P_t yields:

$$s_t^y = (1-\varphi_y) p_{At}^{-\epsilon_y} + \varphi_y (\pi_{t-1}/\pi_t)^{-\epsilon_y} s_{t-1}^y. \tag{A.1f}$$

A.2 Wage Setting

Consider the real wage W_{ht}/P_t of a household member who has set his wage optimally in period t to $\tilde{w}_t = W_{At}/P_t$ and who has not been able to do so again until period $s = 1, 2, \ldots$. This is given by

$$\frac{W_{Nt+s}}{P_{t+s}} = \frac{\prod_{i=1}^{s} \pi_{t+i-1} W_{At}}{\prod_{i=1}^{s} \pi_{t+s} P_t} = \frac{\pi_t}{\pi_{t+s}} \tilde{w}_t,$$

and the demand for his type of labor service equals

$$N_{ht+s} = \left(\frac{(\pi_t/\pi_{t+s})\tilde{w}_t}{w_{t+s}}\right)^{-\epsilon_n} N_{t+s},$$

where w_{t+s} denotes the real wage prevailing in period $t+s$. Accordingly, the Lagrangian for the optimal real wage reads:

$$\begin{aligned} \mathscr{L} =& \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_n)^s \Bigg\{ \frac{(C_{ht+s} - \chi \bar{C}_{t+s})^{1-\eta} - 1}{1-\eta} - \frac{\nu_0}{1+\nu_1} \left[\left(\frac{(\pi_t/\pi_{t+s})\tilde{w}_t}{w_{t+s}}\right)^{-\epsilon_n} N_{t+s} \right]^{1+\nu_1} \\ & + \Lambda_{ht+s} \left[\frac{\pi_t}{\pi_{t+s}} \tilde{w}_t \left(\frac{(\pi_t/\pi_{t+s})\tilde{w}_t}{w_{t+s}}\right)^{-\epsilon_n} N_{t+s} + RMT \right] \Bigg\}. \end{aligned}$$

The first-order condition with respect to $\tilde{w}_t$ is

$$\begin{aligned} 0 = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_n)^s \Bigg\{ & \epsilon_n \nu_0 \tilde{w}_t^{-\epsilon_n(1+\nu_1)-1} \left(\frac{(\pi_t/\pi_{t+s})}{w_{t+s}}\right)^{-\epsilon_n(1+\nu_1)} N_{t+s}^{1+\nu_1} \\ & + (1-\epsilon_n)\Lambda_{ht+s} \tilde{w}_t^{-\epsilon_n} w_{t+s}^{\epsilon_n} \left(\frac{\pi_t}{\pi_{t+s}}\right)^{1-\epsilon_n} N_{t+s} \Bigg\}. \end{aligned}$$

Using $\Lambda_{ht+s} = \Lambda_{t+s}$ this can be arranged to read

$$\tilde{w}_t = \frac{\epsilon_n}{\epsilon_n - 1}\frac{\Delta_{1t}}{\Delta_{2t}}, \tag{A.2a}$$

where

$$\begin{aligned}
\Delta_{1t} &= \nu_0 \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_n)^s \left(\frac{\pi_t \tilde{w}_t}{\pi_{t+s} w_{t+s}}\right)^{-\epsilon_n(1+\nu_1)} N_{t+s}^{1+\nu_1}, \\
&= \nu_0 \left(\frac{\tilde{w}_t}{w_t}\right)^{-\epsilon_n(1+\nu_1)} N_t^{1+\nu_1} + (\beta\varphi_n)\mathbb{E}_t \left(\frac{\pi_t \tilde{w}_t}{\tilde{\pi}_{t+1} w_{t+1}}\right)^{-\epsilon_n(1+\nu_1)} \Delta_{1t+1}, && \text{(A.2b)} \\
\Delta_{2t} &= \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_n)^s \Lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\epsilon_n} \left(\frac{\pi_t}{\pi_{t+s}}\right)^{1-\epsilon_n} N_{t+s}, \\
&= \Lambda_t \left(\frac{\tilde{w}_t}{w_t}\right)^{-\epsilon_n} N_t + (\beta\varphi_n)\mathbb{E}_t \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}}\right)^{-\epsilon_n} \left(\frac{\pi_t}{\pi_{t+1}}\right)^{1-\epsilon_n} \Delta_{2t+1}. && \text{(A.2c)}
\end{aligned}$$

The wage index (2.24) implies

$$W_t^{1-\epsilon_n} = (1-\varphi_n) W_{At}^{1-\epsilon_n} + \varphi_n (\pi_{t-1} W_{t-1})^{1-\epsilon_n}$$

so that the real wage equals

$$w_t^{1-\epsilon_n} = (1-\varphi_n)\tilde{w}_t^{1-\epsilon_n} + \varphi_n \left(\frac{\pi_{t-1}}{\pi_t} w_{t-1}\right)^{1-\epsilon_n}. \tag{A.2d}$$

Finally consider the index

$$\tilde{N}_t^{1+\nu_1} = \int_0^1 N_{ht}^{1+\nu_1} dh,$$

in the families current-period utility function. Using (2.23), this can be written

$$\tilde{N}_t^{1+\nu_1} = N_t^{1+\nu_1} \int_0^1 \left(\frac{W_{ht}}{W_t}\right)^{-\epsilon_n(1+\nu_1)} dh.$$

Let

$$\bar{W}_t^{-\epsilon_n(1+\nu_1)} = \int_0^1 W_{ht}^{-\epsilon_n(1+\nu_1)} dh = (1-\varphi_n)(W_{At})^{-\epsilon_n(1+\nu_1)} + \varphi_n (\pi_{t-1} W_{Nt-1})^{-\epsilon_n(1+\nu_1)}$$

and

$$(s_t^n)^{1+\nu_1} = \left(\frac{\bar{W}_t}{W_t}\right)^{-\epsilon_n(1+\nu_1)} = \left(\frac{\bar{W}_t/P_t}{W_t/P_t}\right)^{-\epsilon_n(1+\nu_1)} = \left(\frac{\bar{w}_t}{w_t}\right)^{-\epsilon_n(1+\nu_1)}.$$

Using the same line of argument employed to derive (A.1f) yields the dynamic equation for the measure of wage dispersion s_t^n:

$$(s_t^n)^{1+\nu_1} = (1-\varphi_n)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\epsilon_n(1+\nu_1)} + \varphi_n \left(\frac{\pi_{t-1} w_{t-1}}{\pi_t w_t}\right)^{-\epsilon_n(1+\nu_1)} (s_{t-1}^n)^{1+\nu_1} \tag{A.2e}$$

so that

$$\tilde{N}_t = s_t^n N_t. \tag{A.2f}$$

Note that we must track the variable $\tilde{N}_t$ in order to compute our welfare measure.

A.3 Dynamics

The full model consists of equations (A.1), (A.2), (2.5), (2.14), (2.19a), (2.28), (2.34), (2.35), (2.36), the resource constraint (2.37), the capital accumulation equation (2.33), and the Taylor rule (2.31). In order to compute our welfare measure we have to add the recursive definitions of V_t^C and V_t^N implied by (2.38). These are

$$V_t^C = \left[\frac{(C_t - \chi C_{t-1})^{1-\eta} - 1}{1-\eta}\right] + \beta \mathbb{E}_t V_{t+1}^C, \tag{A.3a}$$

$$V_t^N = \frac{\nu_0}{1+\nu_1}\tilde{N}_t^{1+\nu_1} + \beta \mathbb{E}_t V_{t+1}^N. \tag{A.3b}$$

A.4 Stationary Solution and Calibration

The model is solved via a second-order approximation of the decision rules at the stationary solution of the deterministic version of the model. This solution follows from the model's equations if we set the shocks equal to $Z_t = 1$, and $G_t = G$ and cancel the time indices.

In the first step we determine v and $\bar{\omega}$. We proceed as Carlstrom and Fuerst (1997, 1998) and employ a log-normal distribution for ϕ with parameters μ_ω and σ_ω. We determine these parameters and the stationary bankruptcy threshold $\bar{\omega}$ from three conditions:

i. We assume a mean of one: $\mathbb{E}(\omega) = \Omega = 1$,

ii. a bankruptcy rate of $\Phi(\bar{\omega}) = 0.00974$ (taken from Carlstrom and Fuerst (1998), p. 590),

iii. and an external finance premium of $\frac{\bar{\omega}}{g(\bar{\omega})} - 1 = r_L = 1.0187^{0.25} - 1$ (also taken from Carlstrom and Fuerst (1998), p. 590)

Given $\bar{\omega}$ we can solve (2.19a) for v.

In the second step we determine the additional discount parameter γ. The stationary versions of (2.28b) and (2.21) imply

$$\gamma = \frac{1 - vg(\bar{\omega})}{vf(\bar{\omega})}.$$

In the third step we solve the stationary wage and price equations. It is immediate from equation (A.1d) that $p_A = 1$ so that equation (A.1f) implies $s^y = 1$ and equation (A.1e) $Y = \tilde{Y}$. Equations (A.1a)-(A.1c) deliver

$$g = \frac{\epsilon_y}{\epsilon_y - 1}, \tag{A.4a}$$

$$\Gamma_1 = \frac{g\Lambda Y \pi^{\epsilon}}{1 - \beta\varphi_y}, \tag{A.4b}$$

$$\Gamma_2 = \frac{\Lambda Y \pi^{\epsilon - 1}}{1 - \beta\varphi_y}. \tag{A.4c}$$

Equation (A.2d) implies $\tilde{w} = w$ so that $\tilde{s}^n = 1$ via (A.2e) and $N = \tilde{N}$ from (A.2f). The stationary values of the auxiliary variables follow from (A.2b) and (A.2c) as

$$\Delta_1 = \frac{N^{1+\nu_1}}{1 - \beta\varphi_n}, \tag{A.4d}$$

$$\Delta_2 = \frac{\Lambda N}{1 - \beta\varphi_n} \tag{A.4e}$$

so (A.2a) implies

$$\nu_0 N^{\nu_1} = \frac{\epsilon_n - 1}{\epsilon_n}\Lambda w. \tag{A.4f}$$

In the fourth step we solve for Y/K. Our assumption with respect to the function Ψ in (2.33) imply $q = 1$ (see (2.5a) and $r_K = 0$ (see (2.5b)) so that equations (2.14b) and (2.28b) can be solved for

$$\frac{Y}{K} = \frac{1 - \beta(1 - \delta)}{\alpha\beta(g/v)}. \tag{A.4g}$$

The production function (2.14c) yields

$$\frac{K}{N} = \left(\frac{Y}{K}\right)^{\frac{1}{\alpha - 1}}. \tag{A.4h}$$

Given N this allows us to compute K, Y, $I = \delta K$. The solution for consumption follows from (2.37):

$$C = Y(1 - g\Phi(\bar{\omega})) - I - G \tag{A.4i}$$

so that Λ is determined by (2.28a):

$$\Lambda = [(1-\chi)C]^{-\eta}. \tag{A.4j}$$

Equation (2.14a) determines the stationary real wage w. We are now able to determine the parameter ν_0 from condition (A.4f) and the auxiliary variables Γ_1, Γ_2, Δ_1 and Δ_2 from (A.4b)-(A.4e).

In the last step we can compute aggregate net worth from equation (2.36):

$$NW = \frac{\tilde{Y}(1 - vg(\bar{\omega}))}{v}, \tag{A.4k}$$

aggregate firm capital from (2.35)

$$X = \frac{NW - \Delta}{1 - \delta + r_Y} \tag{A.4l}$$

and dividends distributed from primary production firms to the household from (2.34)

$$D_f = f(\bar{\omega})\tilde{Y} - X. \tag{A.4m}$$

In our simulations we follow Carlstrom and Fuerst (1998) and set $\Delta = 0$.[8] The stationary values of the life-time utility associated with consumption V^C and working hours V^N equal

$$V^C = \frac{1}{1-\beta}\frac{[(1-\chi)C]^{1-\eta} - 1}{1-\eta}, \tag{A.4n}$$

$$V^N = \frac{1}{1-\beta}\frac{\nu_0}{1+\nu_1}N^{1+\nu_1}. \tag{A.4o}$$

Finally, the stationary version of the Euler equation (2.28c) determines the nominal interest rate

$$Q = \frac{\pi}{\beta}. \tag{A.4p}$$

[8] Carlstrom and Fuerst (1997) assume that Δ_t equals the wage income of entrepreneurs $\alpha_e \tilde{Y}_t$ with α_e close to zero and ignore this term in their 1998 paper.

B Approximation of λ

Note that

$$(1-\lambda)^{1-\eta}V_t^C + \frac{(1-\lambda)^{1-\eta}-1}{(1-\eta)(1-\beta)} = \mathbb{E}_t \sum_{s=0}^{\infty} \frac{(1-\lambda)^{1-\eta}(C_{t+s}-\chi C_{t+s-1})^{1-\eta}-1}{1-\eta}$$

so that condition (2.40) can be written

$$\tilde{V}_t = \tilde{V}_t^C - \tilde{V}_t^N = (1-\lambda)^{1-\eta}V_t^C + \frac{(1-\lambda)^{1-\eta}-1}{(1-\eta)(1-\beta)} - V_t^N. \tag{B.1}$$

This equation can be solve for λ, yielding

$$\lambda = 1 - \left[\frac{1+(1-\eta)(1-\beta)[\tilde{V}_t^C + V_t^N - \tilde{V}_t^N]}{1+(1-\eta)(1-\beta)V_t^C}\right]^{\frac{1}{1-\eta}}.$$

Thus, with $\sigma = 1$, we get

$$\lambda \simeq \lambda(\mathbf{x}) + \lambda_\sigma(\mathbf{x}) + \frac{1}{2}\lambda_{\sigma\sigma}$$

With identical initial conditions $\lambda(\mathbf{x}) = 0$. As shown by Schmitt-Grohé and Uribe (2004a), the first-order effect of the scaling factor σ on the policy functions of the model is nil. As a consequence, $\lambda_\sigma(\mathbf{x}) = 0$. Using this and differentiating (B.1) twice yields the equation (2.41) in the body of the paper.

C Zero Lower Bound

The Taylor rules which we consider must satisfy the non-negativity constraint on the nominal interest rate: $Q_t \geq 1$. Since our solution rests on perturbation methods, we cannot directly take care of this constraint. We, thus, follow Schmitt-Grohé and Uribe (2004a), p.31, who propose to disregard solutions which entail a significant probability to violate this constraint. Assume $Q_t - Q$ is distributed normally with mean zero and variance σ_Q so that $\bar{z} \equiv (1-Q)/\sigma_Q$ is a standard normal random variable. For $\bar{z} = -2.05$ the probability of the event $z \leq \bar{z}$ is 2 percent. Therefor, we disregard solutions for which $\sigma_Q \geq (Q-1)/2.05$.

To determine whether or not a particular monetary policy violates this condition, we must compute the unconditional variance σ_Q^2 of the deviation of the interest factor Q_t from its non-stochastic stationary solution Q.

Let

$$\mathbf{x}_t = \Big[K_t, \quad C_{t-1}, \quad Q_t, \quad w_{t-1}, \quad s^y_{t-1}, \quad s^n_{t-1}, \quad \pi_{t-1}, \quad \omega_{t-1}\Big]'$$

denote the vector of endogenous state variables, $\bar{\mathbf{x}}_t = \mathbf{x}_t - \mathbf{x}$ the deviation of the states from the non-stochastic steady state, and $\mathbf{z}_t = [\ln Z_t, \epsilon^Q_t]'$ the vector of exogenous state variables. The first-order solution of the model is given by

$$\bar{\mathbf{x}}_{t+1} = L^x \bar{\mathbf{x}}_t + L^z \mathbf{z}_t, \tag{C.1}$$

$$\mathbf{z}_{t+1} = \Pi \mathbf{z}_t + \boldsymbol{\epsilon}_{t+1}, \quad \mathbb{E}(\boldsymbol{\epsilon}_{t+1}\boldsymbol{\epsilon}'_{t+1}) = \Sigma^\epsilon = \Omega\Omega'. \tag{C.2}$$

We seek to determine $\Sigma^x \equiv \mathbb{E}(\bar{\mathbf{x}}_t \bar{\mathbf{x}}'_t)$. Since $\mathbf{z}_t$ is a stationary stochastic process and since the eigenvalues of L^x are within the unit circle, Σ^x exists and is independent of the time index t. Multiplying both sides of (C.1) with $\bar{\mathbf{x}}_{t+1}$ yields:

$$\begin{aligned}
\mathbb{E}(\bar{\mathbf{x}}_{t+1}\bar{\mathbf{x}}'_{t+1}) &= \mathbb{E}(L^x\bar{\mathbf{x}}_t + L^z\mathbf{z}_t)(L^x\bar{\mathbf{x}}_t + L^z\mathbf{z}_t)' \\
&= \mathbb{E}(L^x\bar{\mathbf{x}}_t\bar{\mathbf{x}}'_t(L^x)') + \mathbb{E}(L^z\mathbf{z}_t\mathbf{z}'_t(L^z)') + \mathbb{E}(L^x\bar{\mathbf{x}}_t\mathbf{z}'_t(L^z)') + \mathbb{E}(L^z\mathbf{z}_t\bar{\mathbf{x}}'_t(L^x)'), \\
\Sigma^x &= L^x\Sigma^x(L^x)' + L^z\Sigma^z(L^z)' + L^x\Sigma^{xz}(L^z)' + L^z(\Sigma^{xz})'(L^x)'.
\end{aligned}$$

Applying the vec-operator on both sides of the previous equation yields:[9]

$$\operatorname{vec}\Sigma^x = \left(I_{n(x)^2} - L^x \otimes L^x\right)^{-1} \operatorname{vec}\left(L^z\Sigma^z(L^z)' + L^x\Sigma^{xz}(L^z)' + L^z(\Sigma^{xz})'(L^x)'\right). \tag{C.3a}$$

The matrices Σ^{xz} and Σ^z in this expression follow from the same reasoning. Consider $\Sigma^{xz} = \mathbb{E}(\bar{\mathbf{x}}_t\mathbf{z}'_t)$:

$$\begin{aligned}
\Sigma^{xz} &= \mathbb{E}(\bar{\mathbf{x}}_{t+1}\mathbf{z}'_{t+1}) = \mathbb{E}(L^x\bar{\mathbf{x}}_t + L^z\mathbf{z}_t)(\Pi\mathbf{z}_t + \boldsymbol{\epsilon}_{t+1})', \\
&= \mathbb{E}(L^x\bar{\mathbf{x}}_t\mathbf{z}'_t\Pi') + \mathbb{E}(L^z\mathbf{z}_t\mathbf{z}'_t\Pi') + \mathbb{E}(L^x\bar{\mathbf{x}}_t\boldsymbol{\epsilon}'_{t+1}) + \mathbb{E}(L^z\mathbf{z}_t\boldsymbol{\epsilon}'_{t+1}), \\
\Sigma^{xz} &= L^x\Sigma^{xz}\Pi' + L^z\Sigma^z\Pi',
\end{aligned}$$

because the expectation of the terms that involve $\boldsymbol{\epsilon}_{t+1}$ is zero, since $\mathbf{z}_t$ and $\bar{\mathbf{x}}_t$ are predetermined when $\boldsymbol{\epsilon}_{t+1}$ is realized. Therefore,

$$\operatorname{vec}\Sigma^{xz} = \left(I_{n(x)n(z)} - \Pi \otimes L^x\right)^{-1} \operatorname{vec}\left(L^z\Sigma^z\Pi'\right). \tag{C.3b}$$

[9] The respective rule is $\operatorname{vec}(ABC) = (C' \otimes A)\operatorname{vec} B$, where $\otimes$ denotes the Kronecker product of the matrices C' and A. Since the eigenvalues of $C' \otimes A$ are equal to the product of the eigenvalues of C' and A, the eigenvalues of $L^x \otimes L^x$ are within the unit circle and $I - L^x \otimes L^x$ is invertible. See Lütkepohl (2005), p. 661-662 for these results.

Finally:

$$\begin{aligned}
\Sigma^z &\equiv \mathbb{E}(\mathbf{z}_{t+1}\mathbf{z}'_{t+1}) = \mathbb{E}(\Pi\mathbf{z}_t + \boldsymbol{\epsilon}_{t+1})(\Pi\mathbf{z}_t + \boldsymbol{\epsilon}_{t+1})', \\
&= \mathbb{E}(\Pi\mathbf{z}_t\mathbf{z}'_t\Pi') + \mathbb{E}(\boldsymbol{\epsilon}_{t+1}\boldsymbol{\epsilon}'_{t+1}) + \mathbb{E}(\Pi\mathbf{z}_t\boldsymbol{\epsilon}'_{t+_1}) + \mathbb{E}(\boldsymbol{\epsilon}_{t+1}\mathbf{z}'_t\Pi'), \\
\Sigma^z &= \Pi\Sigma^z\Pi' + \Sigma^\epsilon
\end{aligned}$$

so that

$$\operatorname{vec}\Sigma^z = \left(I_{n(z)^2} - \Pi \otimes \Pi)^{-1}\right)^{-1} \operatorname{vec}\Sigma^\epsilon. \tag{C.3c}$$

Equations (C.3) allow us to compute σ_Q as the square root of the third diagonal element of Σ^x, given the model's first order solution L^x and L^z.